KF (cont), Local Planners Fast Robots, ECE4160/5160, MAE 4190/5190

E. Farrell Helbling, 3/11/25



Class Action Items

- Lab 6 begins this week, looking at orientation control of your robot.
 - Good example: either of your TAs (Daria has a great example of a succinct report)
 - Good example with DMP: Stephan Wagner
- Please start working with a lab partner. We are entering the robot skills part of this class, it is highly likely that your robot will break. When this happens, borrow your friends robot. Get the other working in the meantime so you always have a working robot.
- Regrade requests, submit a comment on your grade on Canvas. You have one week after grades are posted to do so. I will collate all comments, TAs will respond as needed after the week.
 - We are known to make mistakes (there are many reports to read), but be sure you need points back, we reserve the right to further deduct points if we find an error in our grading scheme.
 - Lab 1 and Lab 2 regrade requests will close on Sunday midnight.
- Continuing with Kalman Filtering today for your Lab 7, local planners



Kalman Filter

observations, assuming that posterior and prior beliefs are Gaussian



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Incorporate uncertainty to get better estimates based on both inputs and





covariance matrices:









- Define A, B, and C matrices
 - System ID on step response



$$F = ma = m\ddot{x}$$
$$F = u - d\dot{x}$$
$$m\ddot{x} = u - d\dot{x}$$
$$\dddot{x} = \frac{u}{m} - \frac{d}{m}\dot{x}$$

What are d and m?







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$$F = u - d\dot{x}$$
$$m\ddot{x} = u - d\dot{x}$$
$$\dddot{x} = \frac{u}{m} - \frac{d}{m}\dot{x}$$

What are d and m?

At constant speed, we can find d:

$$0 = \frac{u}{m} - \frac{d}{m}\dot{x} \qquad d = \frac{u}{\dot{x}}$$







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At constant speed, we can find d:

$$0 = \frac{u}{m} - \frac{d}{m}\dot{x} \qquad d = \frac{u}{\dot{x}}$$

(assume u=1 for now)
$$d \approx \frac{1}{2000 \text{ mm/s}}$$





State space equations

$$\begin{bmatrix} \dot{x} \\ \dot{x} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & -\frac{d}{m} \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix}$$

$$C = \begin{bmatrix} -1 & 0 \end{bmatrix}$$



$$F = ma = m\ddot{x}$$
$$F = u - d\dot{x}$$
$$m\ddot{x} = \frac{u - d\dot{x}}{d}$$
$$\dddot{x} = \frac{u - d\dot{x}}{m}$$

1st order syste

$$\frac{dy(t)}{dt} + \frac{1}{\tau}y(t)$$

$$\frac{1}{\tau}y(t) = 1 - e^{-\frac{t}{\tau}}$$

What are d and m?

Use the rise time to determine m

$$\dot{v} = \frac{u}{m} - \frac{d}{m}v$$

$$v = 1 - e^{-\frac{d}{m}t_{0.9}} \quad \ln(1 - v) = -\frac{d}{m}t_{0.9}$$

$$m = \frac{-dt_{0.9}}{\ln(1 - 0.9)}$$









State space equations

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1st order syste

$$dy(t)$$
 1
 $dt(t)$
 $+$
 dt
 τ

 Unit step respondent

 $y(t) = 1 - e^{-\frac{t}{\tau}}$

What are d and m?

Use the rise time to determine m

$$\dot{v} = \frac{u}{m} - \frac{d}{m}v$$

$$v = 1 - e^{-\frac{d}{m}t_{0.9}} \qquad \ln(1 - v) = -\frac{d}{m}t_{0.9}$$

$$m = \frac{-dt_{0.9}}{\ln(1 - 0.9)} = \frac{-0.0005 \cdot 1.9}{\ln(0.1)} = 4.1258 \cdot 10^{-4}$$







State space equations

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$$F = ma = m\ddot{x}$$
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$$\ddot{x} = \frac{u}{m} - \frac{d}{m}\dot{x}$$

What are d and m?

At steady state (constant speed) we can find d $d = \frac{u}{\dot{x}} \approx 0.0005$ (assume u=1 for now)

We can use the rise time to find m

$$m = \frac{-dt_{0.9}}{\ln(1 - 0.9)} \approx 4.1258 \cdot 10^{-4}$$





State space equations

$$\begin{bmatrix} \dot{x} \\ \dot{x} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & -\frac{d}{m} \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix}$$

$$C = \begin{bmatrix} -1 & 0 \end{bmatrix}$$



- We have A, B, C
- Discretize the A and B matrices
 - x(n+1) = x(n) + dx
 - $dx/dt = Ax + Bu \iff dx = dt$
 - x(n + 1) = x(n) + dt(Ax(n) + dt)
 - $x(n+1) = (I + dt \cdot A)x(n) + dt$ A_d
 - dt is our sampling time (0.130s)
- Rescale from unity input to actual input

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$$\frac{dt}{Bu}$$

$$Bu$$

$$Bu$$

$$Bd$$

State space equations $\begin{vmatrix} 0 & 1 \\ 0 & -\frac{d}{2} \end{vmatrix} \begin{bmatrix} x \\ \dot{x} \end{bmatrix} + \begin{vmatrix} 0 \\ 1 \\ \frac{1}{2} \end{vmatrix}$ m $\lfloor m \rfloor$ $C = [-1 \ 0]$



Lab 7: Kalman Filter Implement the Kalman Filter

Kalman Filter (
$$\mu(t - 1), \Sigma(t - 1), u(t), z(t)$$
)
1. $\mu_p(t) = A\mu(t - 1) + Bu(t)$
2. $\Sigma_p(t) = A\Sigma(t - 1)A^T + \Sigma_u$
3. $K_{KF} = \Sigma_p(t)C^T(C\Sigma_p(t)C^T + \Sigma_z)^{-1}$
4. $\mu(t) = \mu_p(t) + K_{KF}(z(t) - C\mu_p(t))$
5. $\Sigma(t) = (I - K_{KF}C)\Sigma_p(t)$
6. Return $\mu(t)$ and $\Sigma(t)$

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Next, determine measurement and process noise

```
f kf(mu,sigma,u,y):
```

```
mu_p = A.dot(mu) + B.dot(u)
sigma_p = A.dot(sigma.dot(A.transpose())) + Sigma_u
sigma_m = C.dot(sigma_p.dot(C.transpose())) + Sigma_z
kkf_gain = sigma_p.dot(C.transpose().dot(np.linalg.inv(sigma_m)))
y_m = y-C.dot(mu_p)
mu = mu_p + kkf_gain.dot(y_m)
sigma=(np.eye(2)-kkf_gain.dot(C)).dot(sigma_p)
```

return mu,sigma



Lab 7: Kalman Filter Implement the Kalman Filter

• Measurement noise

•
$$\Sigma_z = [\sigma_3^2]$$

• $\sigma_3^2 = (20 \text{mm})^2$

Process noise (dependent on sampling rate)

$$\Sigma_{u} = \begin{bmatrix} \sigma_{1}^{2} & 0 \\ 0 & \sigma_{2}^{2} \end{bmatrix}$$
 Sample

- Trust in modeled position:
 - Pos_{stddev} after 1s: $\sqrt{10^2 \cdot \frac{1}{0.13}} = 27.7 \text{mm}$
- Trust in modeled speed:
 - Speed_{stddev} after 1s: $\sqrt{10^2 \cdot \frac{1}{0.13}} = 27.7 \text{mm/s}$







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Lab 7: Kalman Filter Implement the Kalman Filter

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5. $\Sigma(t) = (I - K_{KF}C)\Sigma_p(t)$
6. Return $\mu(t)$ and $\Sigma(t)$

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Finally, determine your initial state mean and covariance

$$\mu(t-1)$$
$$\Sigma(t-1)$$

f kf(mu,sigma,u,y):

```
mu_p = A.dot(mu) + B.dot(u)
    sigma_p = A.dot(sigma.dot(A.transpose())) + Sigma_u
    sigma_m = C.dot(sigma_p.dot(C.transpose())) + Sigma_z
    kkf_gain = sigma_p.dot(C.transpose().dot(np.linalg.inv(sigma_m)))
    y_m = y-C.dot(mu_p)
    mu = mu_p + kkf_gain.dot(y_m)
sigma=(np.eye(2)-kkf_gain.dot(C)).dot(sigma_p)
```

return mu,sigma









- Define A, B, and C matrices
 - System ID on step response
- Sanity check
 - Run virtual kalman filter on data from Lab 5 PID
 - What is your initial state, and how confident are you in it?
 - How much trust do you put in your model versus your sensor values?
 - Experiment
 - Put less trust in the model
 - Put less trust in the sensor
 - Start with a bad initial estimate
 - Our dynamic model is a bad estimate for the static robot



Linear System Review

- Linear system: $\dot{x} = Ax$
- Solution: $x(t) = e^{At}x(0)$
- Eigenvectors: $T = \begin{bmatrix} \xi_1 & \xi_2 & \dots & \xi_n \end{bmatrix}$

Eigenvalues: $D = \begin{bmatrix} \lambda_1 \\ \lambda_2 \end{bmatrix}$

>>[T,D] = eig(A)

- Linear Transform: AT = TD
- Solution: $e^{At} = e^{TDT^{-1}t}$
- Mapping from x to z to x: $x(t) = Te^{Dt}T^{-1}x(0)$
- Stability in continuous time: $\lambda = a + ib$, stable iff a < 0Optimal Observer (KF): sensor/model noise



- Discrete time: $x(k + 1) = \tilde{A}x(k)$, where $\tilde{A} = e^{A\Delta t}$
- Stability in discrete time: $\tilde{\lambda}^n = R^n e^{in\theta}$, stable iff R
- Nonlinear systems: $\dot{x} = f(x)$

• Linearization: $\frac{Df}{Dx}$

- Controllability: $\dot{x} = (A BK)x$ |>>rank(ctrb(A, B))
- Reachability
- **Controllability Gramian**
- Pole Placement |>>place(A,B,poles)
- **Optimal Control (LQR)** |>>LQR (A, B, Q, R)



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What we've covered so far...

- Configuration space and transformations
- Data types •
- Sensors
- Actuators/Motors
- Wiring/EMI •
- Control \bullet
 - State space models
 - PID/LQR control
 - Observers ullet
- Deterministic vs. Probabilistic Robots
 - Bayes Theorem

- Next up....





Navigation and Planning

Slides adapted from Vivek Thangavelu



- avoiding collisions
- How do you get to your goal?
 - Can you see your goal?
 - Do you have a map?
 - Are obstacles unknown or dynamic?
 - Does it matter how fast you get there?
 - Does it matter how smooth the path is?
 - How much compute power do you have?
 - How precise and accurate is your motion control?
 - What sensors do you have available?
 - etc.

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Problem: Find the path in the workspace from an initial location to a goal location, while



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- **Problem:** Find the path in the workspace from an initial location to a goal location, while avoiding collisions
- Assumption: A good map for navigation exists
- **Global navigation** •



Local navigation





Break the problem down: localization, map building, path planning





Next module on navigation

Local planners

- Global localization and planning
 - Map representations
 - Continuous
 - Discrete
 - Topological
 - Maps as graphs
 - Graph search algorithms
 - Breadth first search
 - Depth first search
 - Dijkstras
 - A*





Local Planners



Local path planning/ obstacle avoidance

- - Can be based on a local map

 - Often implemented as a separate task Runs at a much faster rate than the global planner
 - 3 examples:
 - BUG algorithms
 - Vector Field Histogram (VFH)
 - Dynamic Window Approach (DWA)

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Use goal position, recent sensor readings, and relative position of robot to goal



Dashed blue spline is global path: a) Green spline is ideal local Fig. 1. path; b) Red spline is actual local path





Bug algorithms

- Uses local knowledge and the direction and distance to the goal
- Basic idea
 - Follow the contour of obstacles until you see the goal
 - State 1: seek goal
 - State 2: follow wall
- Different Variants: Bug0, Bug1, Bug2
- Advantages
 - Super simple
 - No global map
 - Completeness
- Disadvantages
 - Suboptimal





- Sensor Assumptions
 - Direction to the goal
 - Detect walls lacksquare
- Algorithm
 - Go towards goal
 - Follow obstacles until you can go towards goal again
 - Loop

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- Sensor Assumptions
 - Direction to the goal
 - Detect walls
 - Odometry
- Algorithm
 - Go towards goal
 - Follow obstacles and remember how close you got to the goal
 - Return to the closest point, loop

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- Sensor Assumptions
 - Direction to the goal
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Bug 1 - formally

- Sensor Assumptions
 - Direction to the goal
 - Detect walls
 - Odometry
- Lower bound traversal? d
- Upper bound traversal? $d + 1.5\Sigma(P_n)$
- Pros?
 - If a path exists, it returns in finite time
 - It knows if none exist!

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- Sensor Assumptions
 - Direction to the goal
 - Detect walls lacksquare
 - Odometry \bullet
 - Original vector to the goal
- Algorithm
 - Go towards goal on the vector
 - Follow obstacles until you are back on the vector (and closer to the obstacle)
 - Loop

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- Sensor Assumptions
 - Direction to the goal
 - Detect walls
 - Odometry lacksquare
 - Original vector to the goal
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 - Direction to the goal
 - Detect walls \bullet
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- Algorithm
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What is faster, right- or left- wall following?

Battle of the bugs (1 vs 2)

Bug 1 Layout 1

Bug 1 Layout 1

https://www.youtube.com/watch?v=T2PVaKyxMmY



Battle of the bugs (1 vs 2)

Exhaustive search

Bug 1 Layout 2

Greedy search

Bug 2 Layout 2

https://www.youtube.com/watch?v=T2PVaKyxMmY



Bug algorithms

- Uses local knowledge and the direction and distance to the goal
- Basic idea
 - Follow the contour of obstacles until you see the goal
 - State 1: seek goal
 - State 2: follow wall
- Different Variants: Bug0, Bug1, Bug2

- The robot motion behavior is reactive
- Issues if the instantaneous sensor readings do not provide enough information or are noisy





Vector Field Histograms

- VFH creates a local map of the environment around the robot populated by "relatively" recent sensor readings
- Build a local 3D grid map reduce to a 1-DOF histogram
- Planning

 - Find all openings large enough for robot to pass Choose the one with the lowest cost, G
 - G = a*goal_direction + b*orientation +c*prev_direction





Vector Field Histograms

- VFH creates a local map of the environment around the robot populated by "relatively" recent sensor readings
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 - Find all openings large enough for robot to pass Choose the one with the lowest cost, G • G = a*goal_direction + b*orientation +c*prev_direction

 - VFH+: incorporate kinematics
- Limitations
 - Does not avoid local minima
 - Not guaranteed to reach goal







Dynamic Window Approach

- Search in the velocity space (robot moves in circular arcs) Takes into account robot acceleration and update rates
- A dynamic window, V_d , is the set of all tuples (v_d , ω_d) that can be reached
- Admissable velocities, V_a, include those where the robot can stop before collision
- The search space is then $V_r = V_s \cap V_a \cap V_d$
- Cost function: $G(v,\omega) = \sigma(\alpha \cdot heading(v,\omega) + \beta \cdot dist(v,\omega) + \gamma \cdot velocity(v,\omega))$



Figure 4. Velocity space

90 deg/sec





http://www4.cs.umanitoba.ca/~jacky/ Teaching/Courses/74.795-LocalVision/ ReadingList/fox97dynamic.pdf



Local Planners

- Bug algorithms
 - Inefficient but can be exhaustive
- Vector Field Histograms
 - Takes into account probabilistic sensor measurements
- Vector Field Histograms+
- Dynamic window approach
 - Takes into account robot dynamics

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Takes into account probabilistic sensor measurements and robot kinematics



Global localization



Next module on navigation

Local planners

Global localization and planning

- Map representations
 - Continuous
 - Discrete
 - Topological
- Maps as graphs
 - Graph search algorithms
 - Breadth first search
 - Depth first search
 - Dijkstras
 - A*





Break the problem down: localization, map building, path planning

Localization Problem

Position Tracking

- Initial robot pose is known
- Either deterministically (odometry) or through Bayesian statistic (motion and sensor models)
- It is a "local" problem, as the uncertainty is local (often small) and confined to a region near the robot's true pose

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Global Localization

- Initial robot pose is unknown
- Need to estimate position from scratch
- A more difficult "global" problem, where you cannot assume boundedness in pose error

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