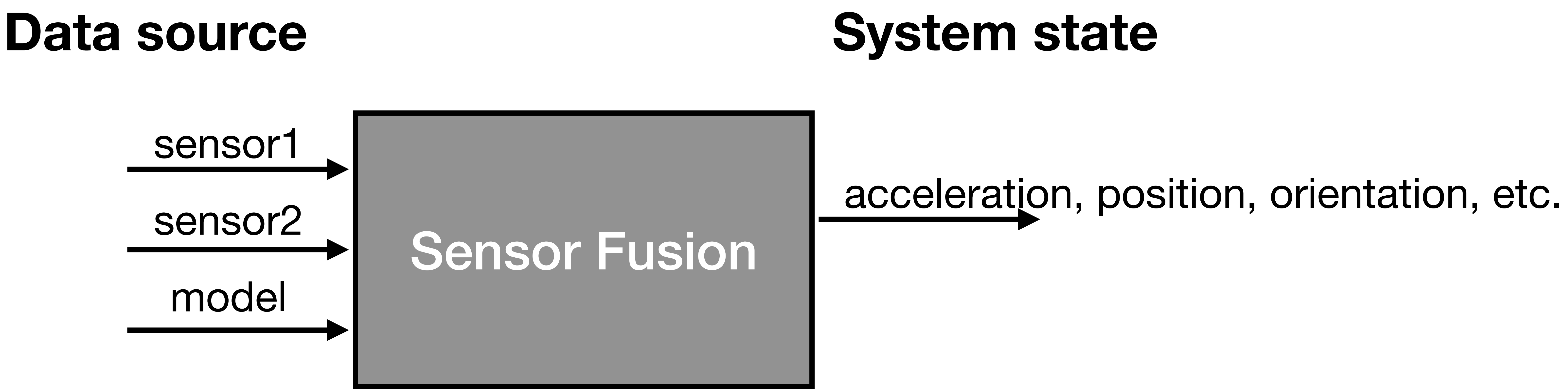
Sensors II

Fast Robots, ECE4160/5160, MAE 4190/5190

E. Farrell Helbling, 1/29/26
Slides adapted from Prof. Kirstin Petersen

Sensor Fusion

- Combine two or more data sources in a way that generates “better” understanding of the system
  - More consistent, accurate, and dependable signal over time

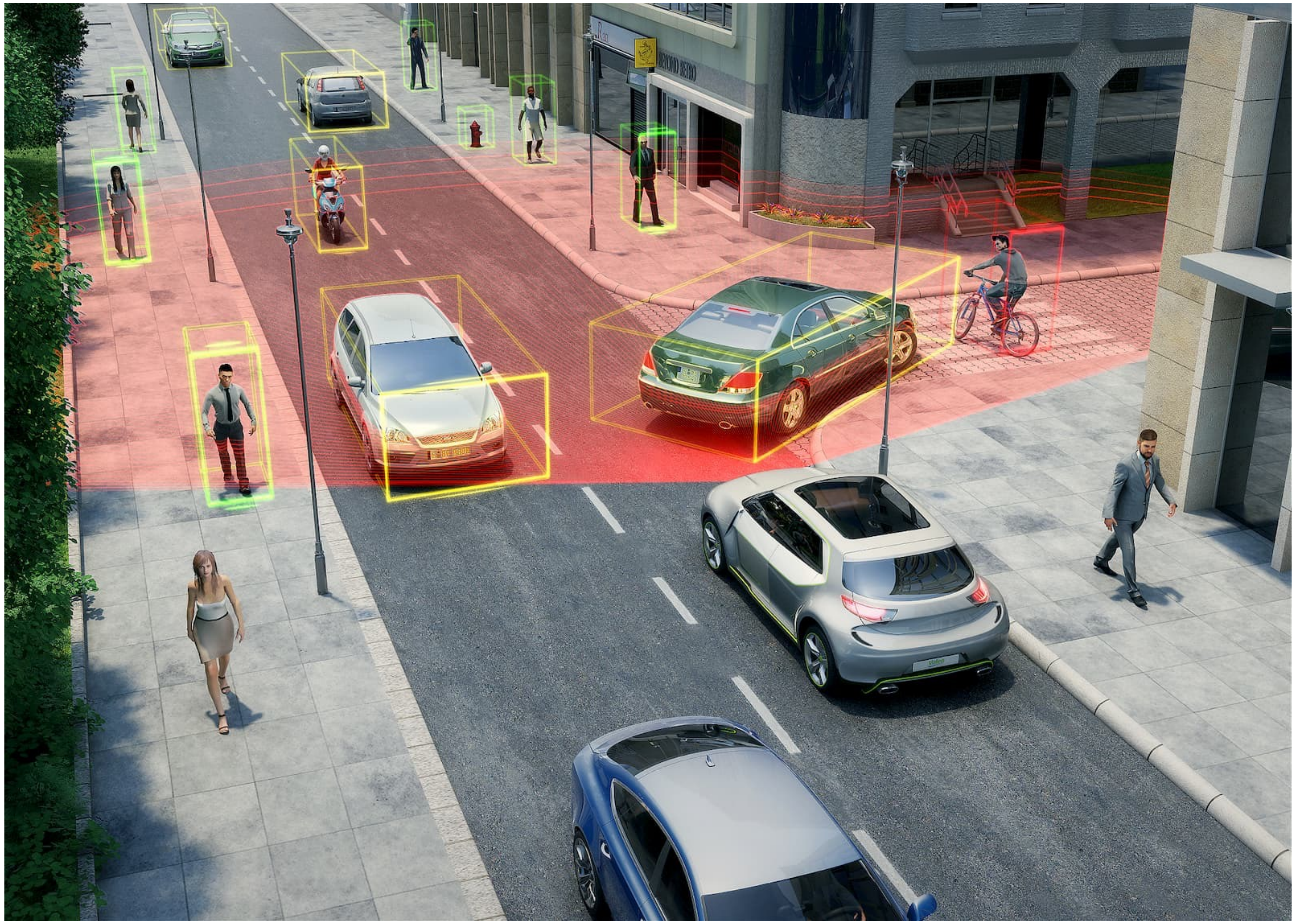


Sensor Fusion

- Combine two or more data sources in a way that generates “better” understanding of the system
  - More consistent, accurate, and dependable signal over time

- Responsibilities:
  - Self-awareness (where am I? what am I doing? what is my current state?
  - Situational awareness (detection/ tracking)

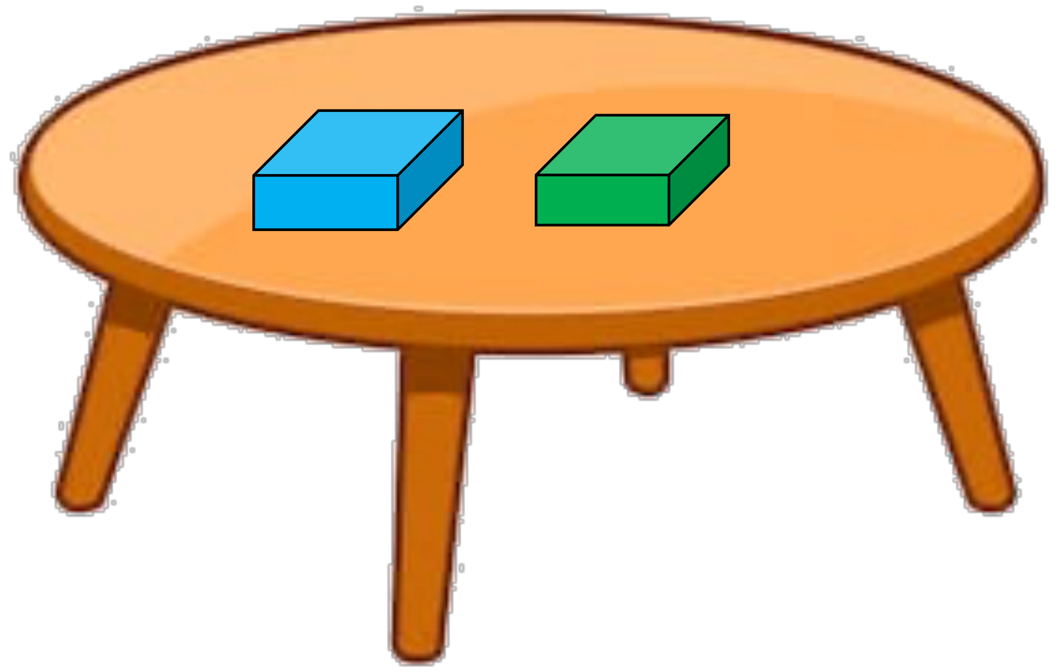
Situational awareness

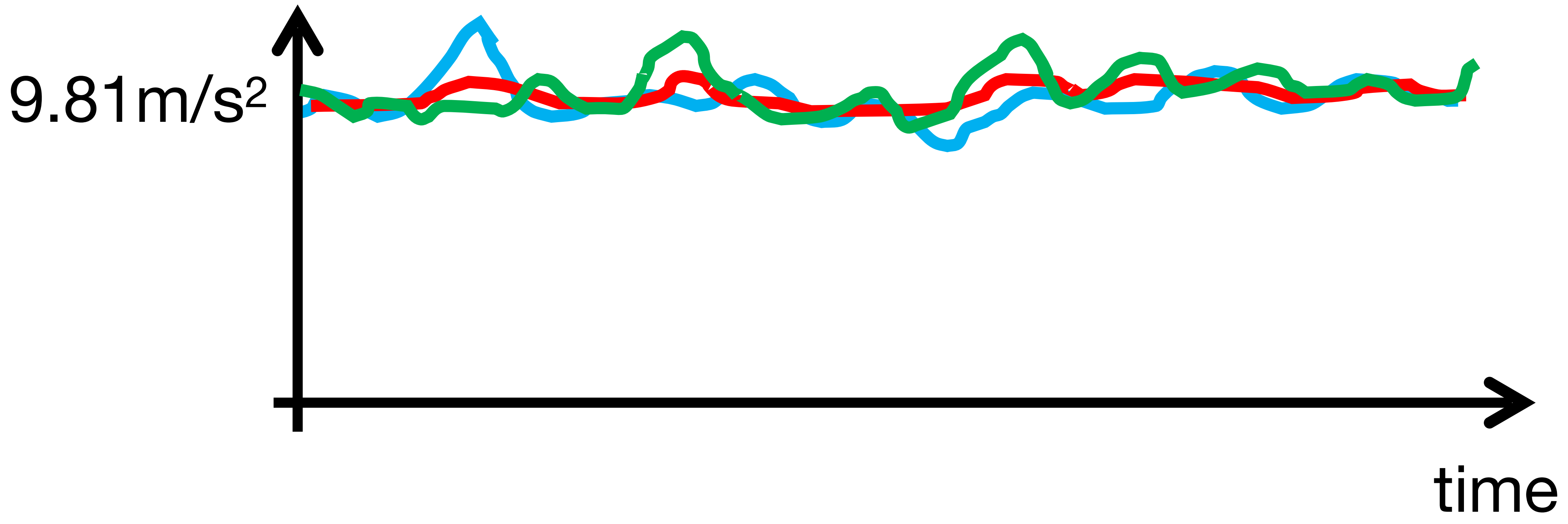


Valeo's LIDAR

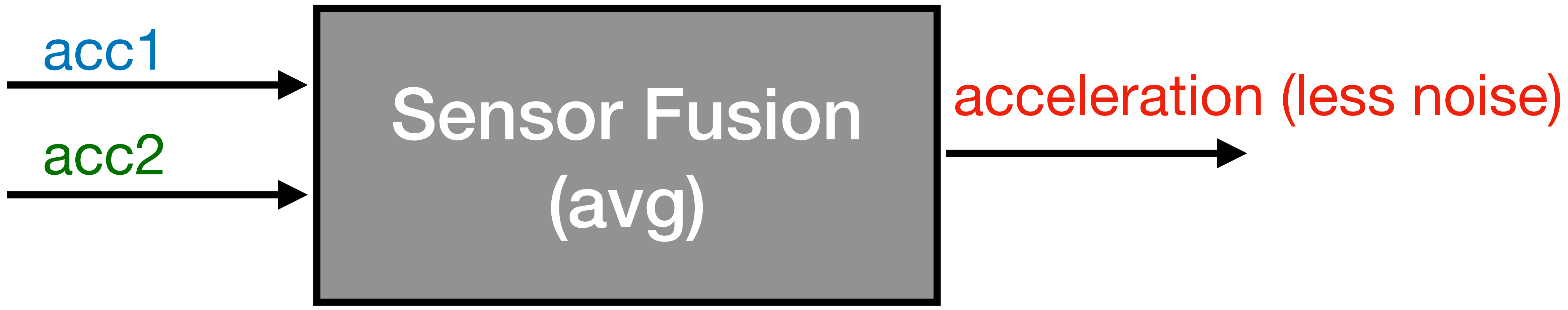
Sensor Fusion

- Increase the quality of the data: less noise, uncertainty, deviation



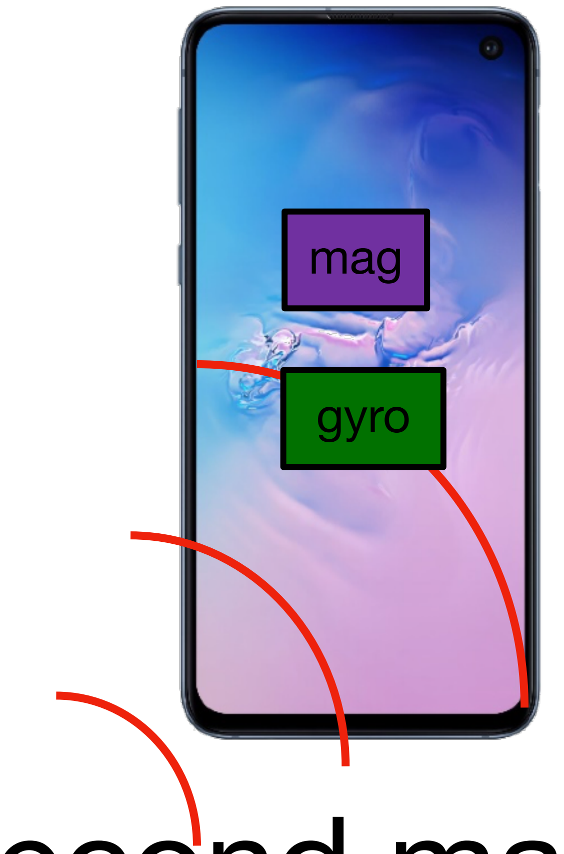


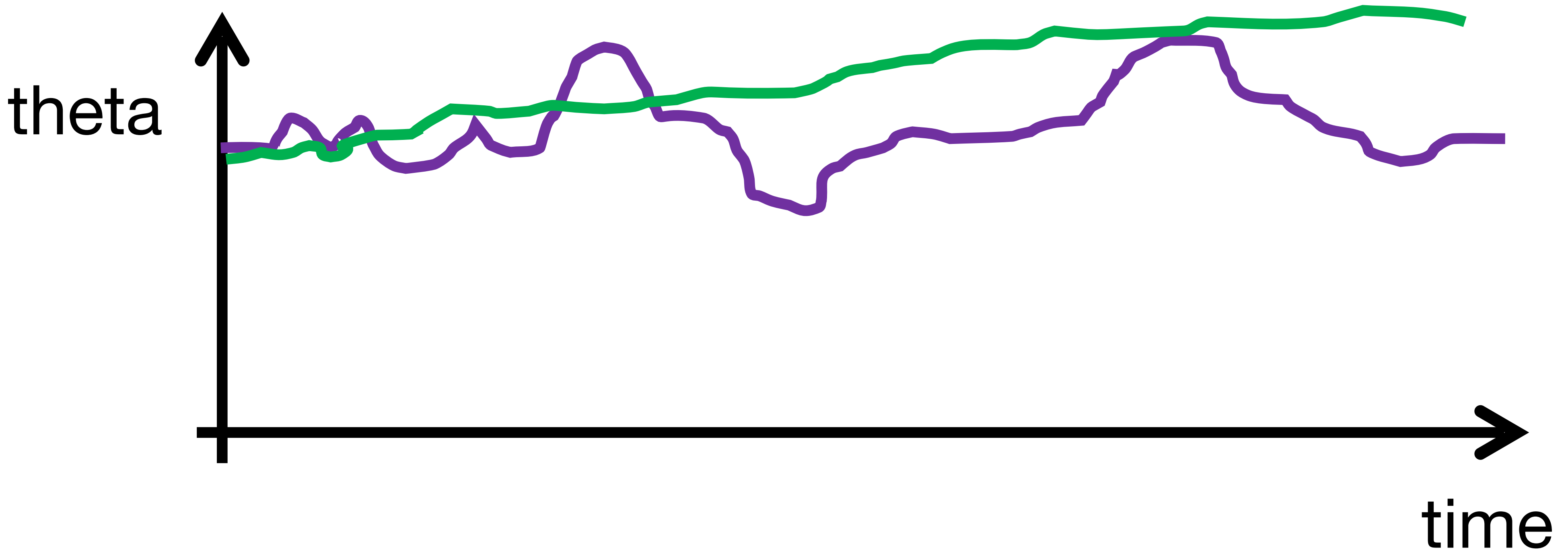
- Adding sensors lowers noise:
    - $n = 1/\sqrt{N}$
    - Only true if the noise is not correlated!



Sensor Fusion

- Increase the quality of the data: less noise, uncertainty, deviation

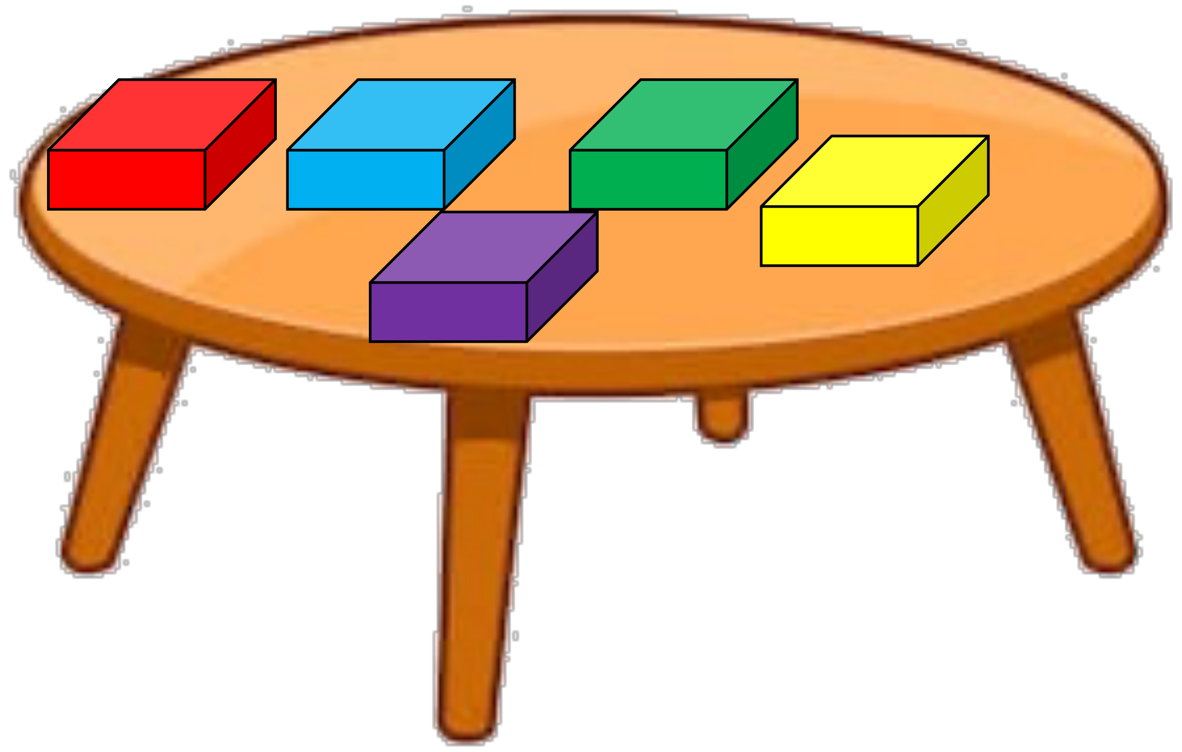


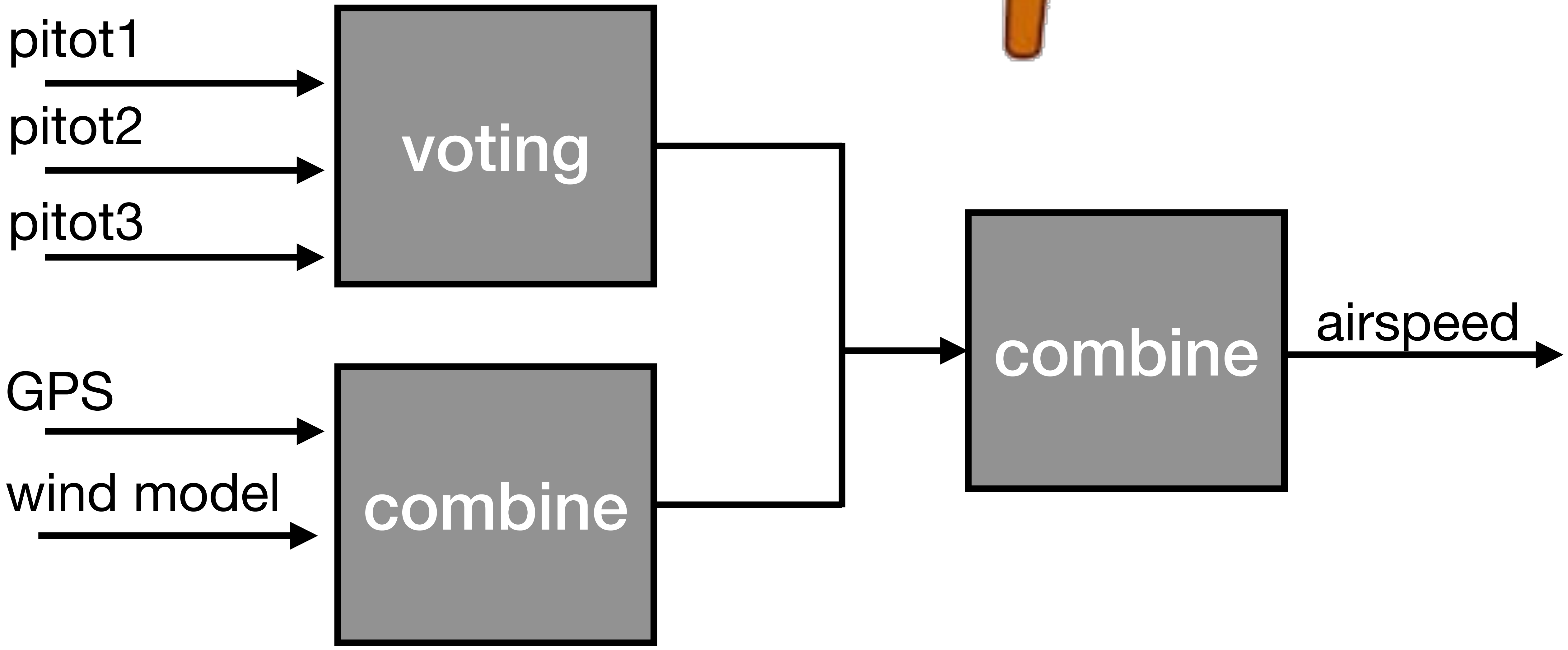


- Add a second mag?
- Move the sensor away from the field
- Low pass filter
- Fuse the mag data with the gyroscope data

Sensor Fusion

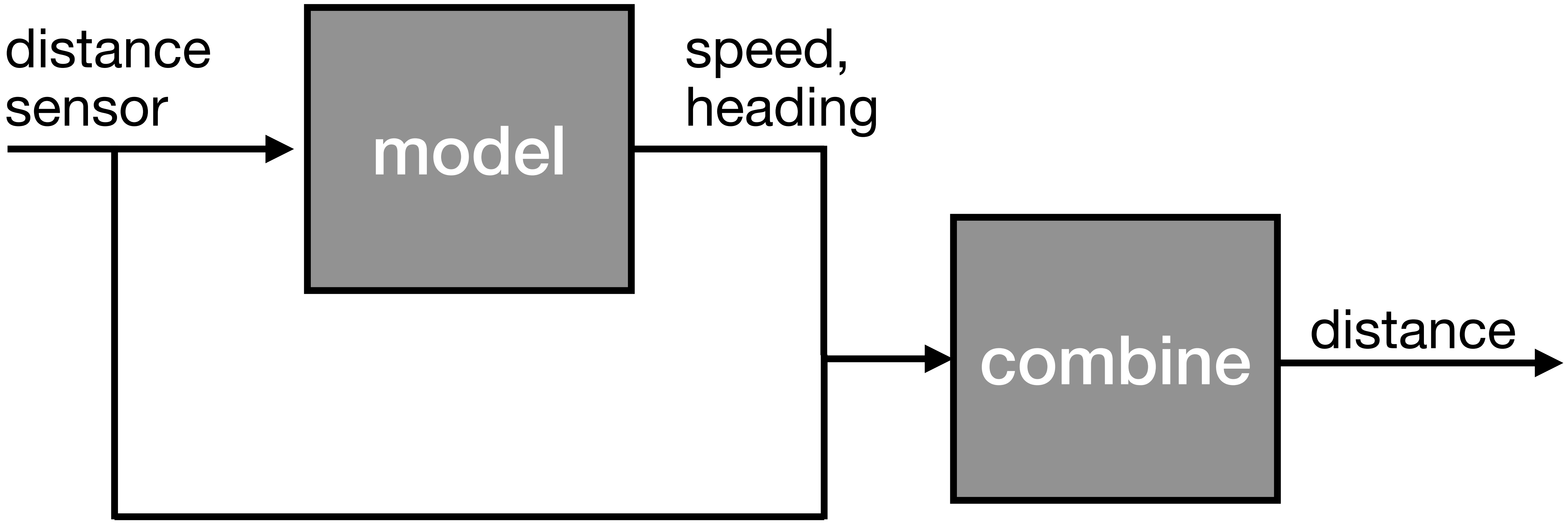
- Increase the quality of the data: less noise, uncertainty, deviation
- Increase data reliability





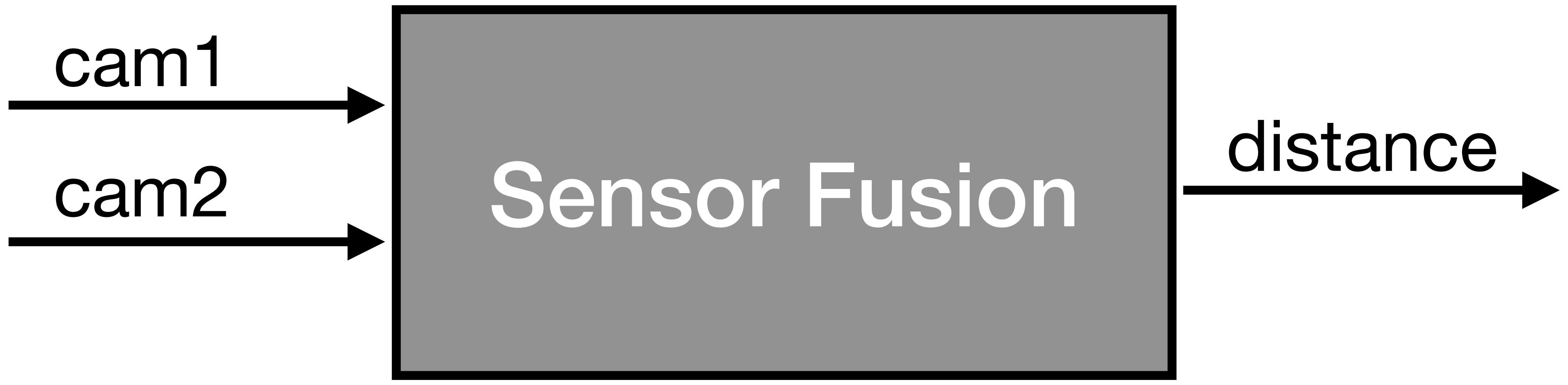
Sensor Fusion

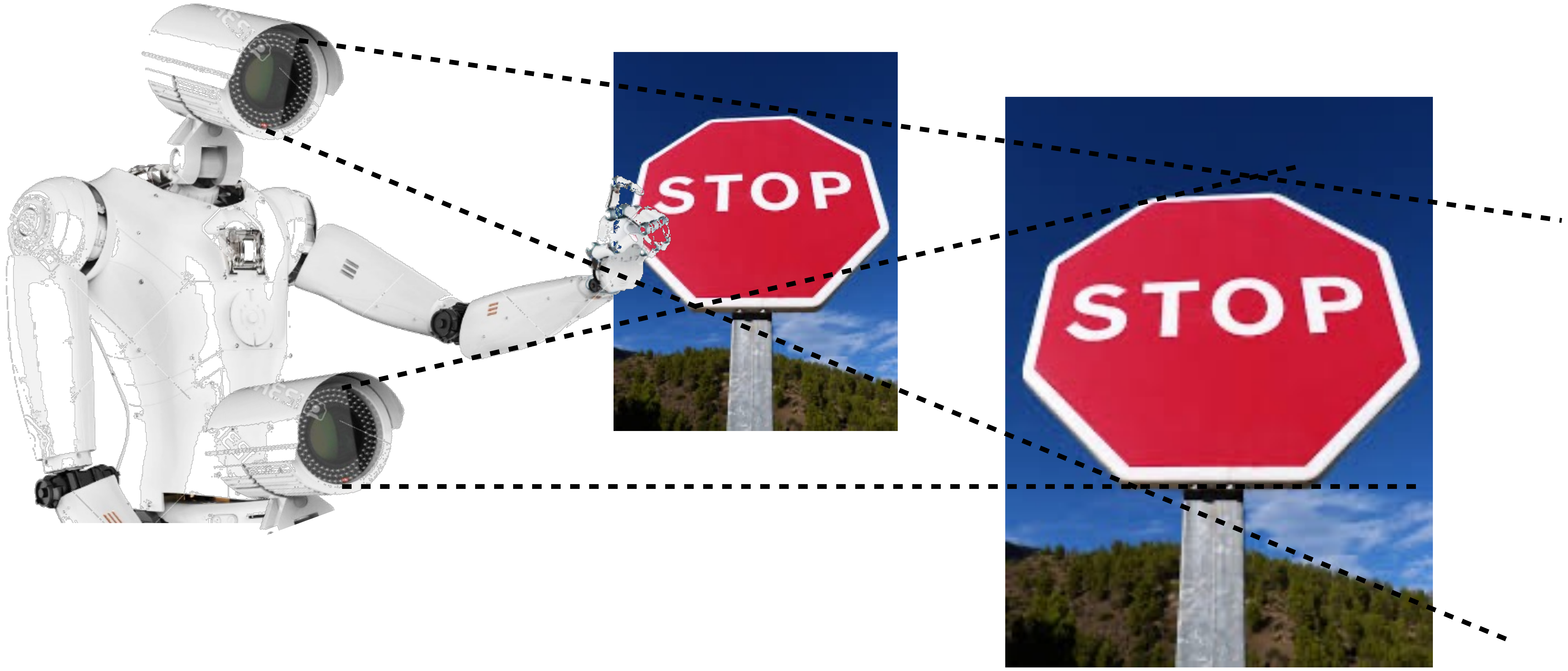
- Increase the quality of the data: less noise, uncertainty, deviation
- Increase data reliability



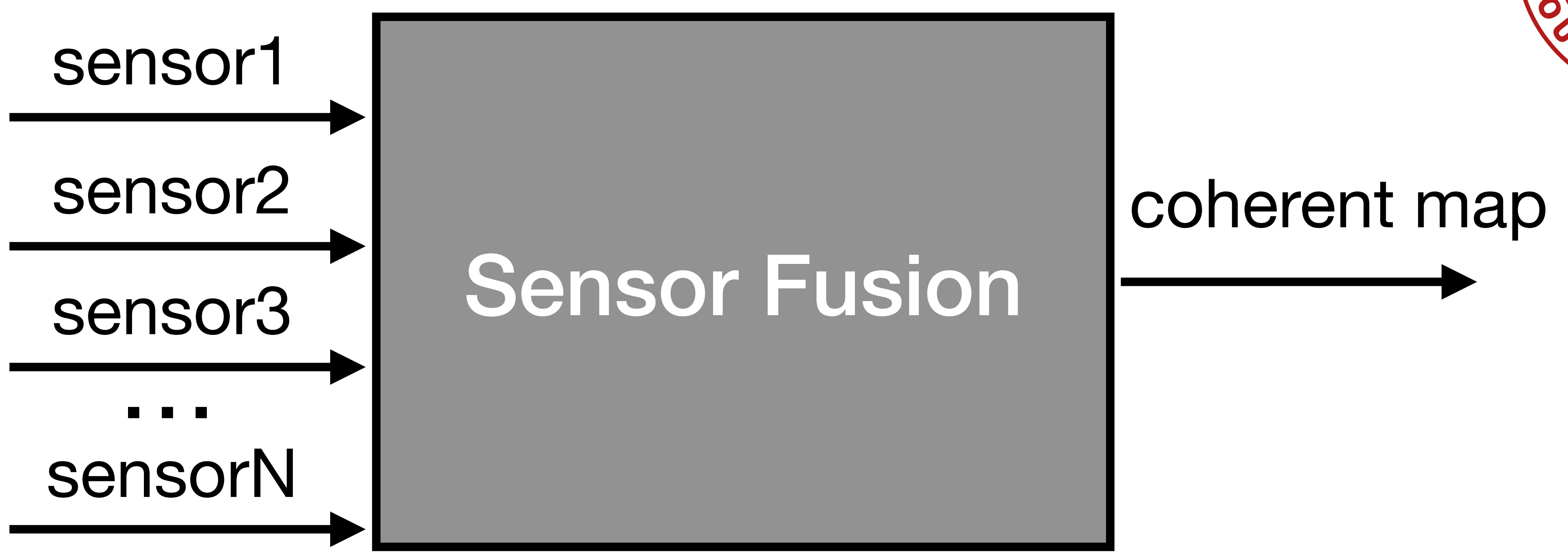
Sensor Fusion

- Increase the quality of the data: less noise, uncertainty, deviation
- Increase data reliability
- Measure unmeasured states

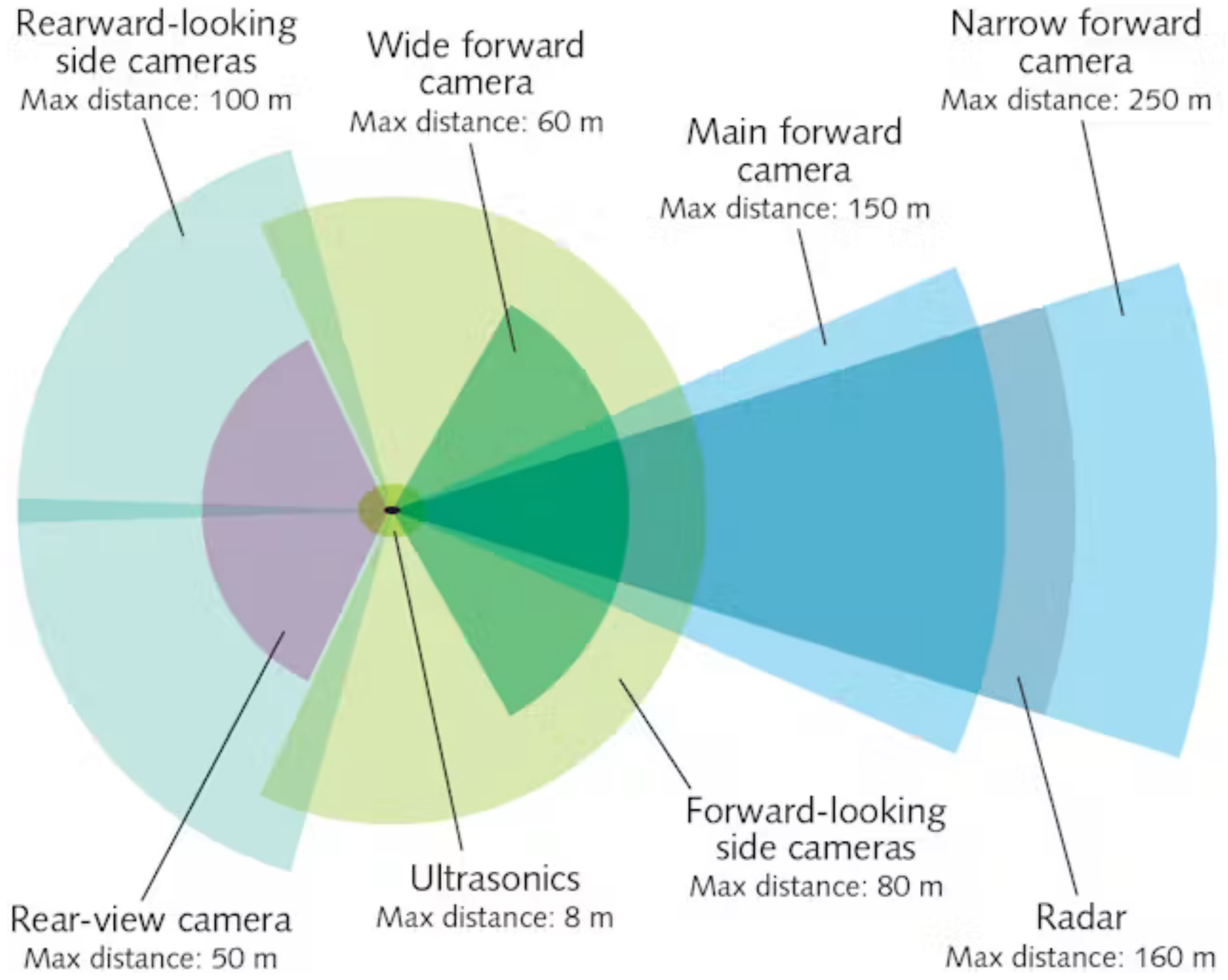




Sensor Fusion



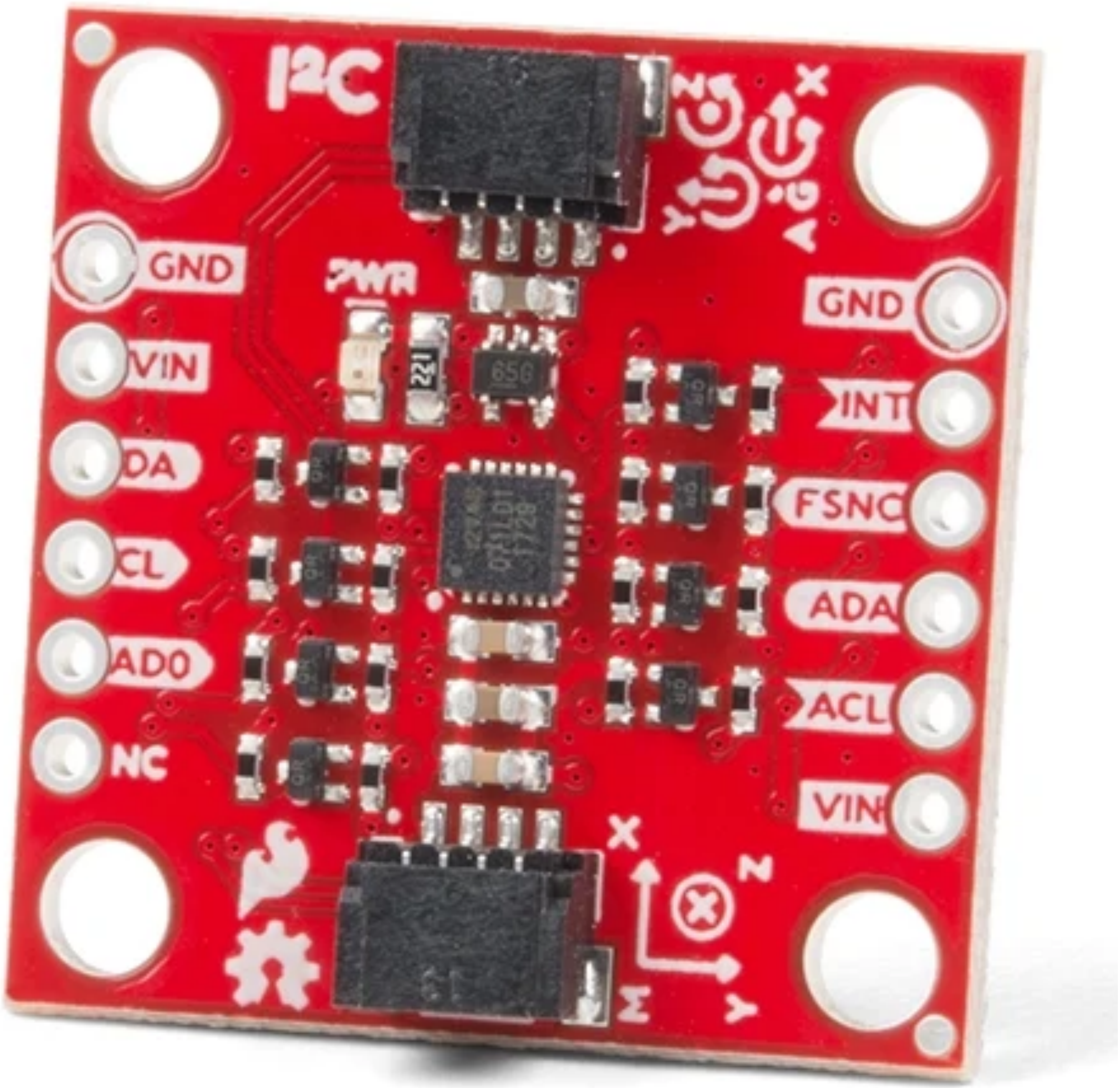
- Increase the quality of the data:
  - less noise, uncertainty, deviation
- Increase data reliability
- Measure unmeasured states
- Increase coverage area



Sources and references

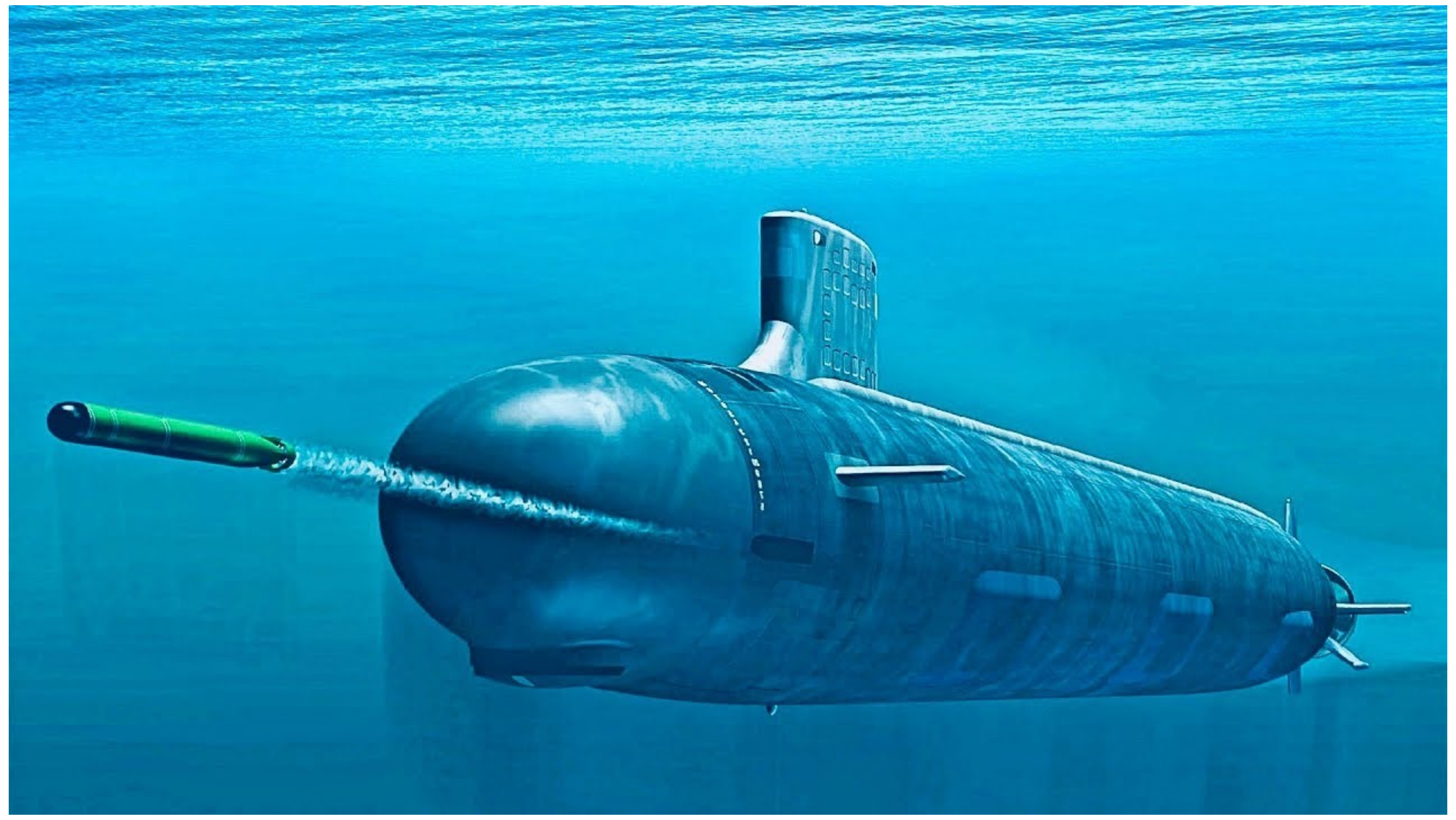
- http://www.cs.cmu.edu/~rasc/Download/AMRobots4.pdf
- https://www.ti.com/lit/ug/sbau305b/sbau305b.pdf?ts=1599417595209&ref_url=https%253A%252F%252Fwww.google.com%252F
- https://hmc.edu/lair/ARW/ARW-Lecture01-Odometry.pdf
- Matlab Tech Talks on Sensor Fusion (https://www.youtube.com/watch?v=6qV3YjFppuc)
- Prof. Kirstin Petersen

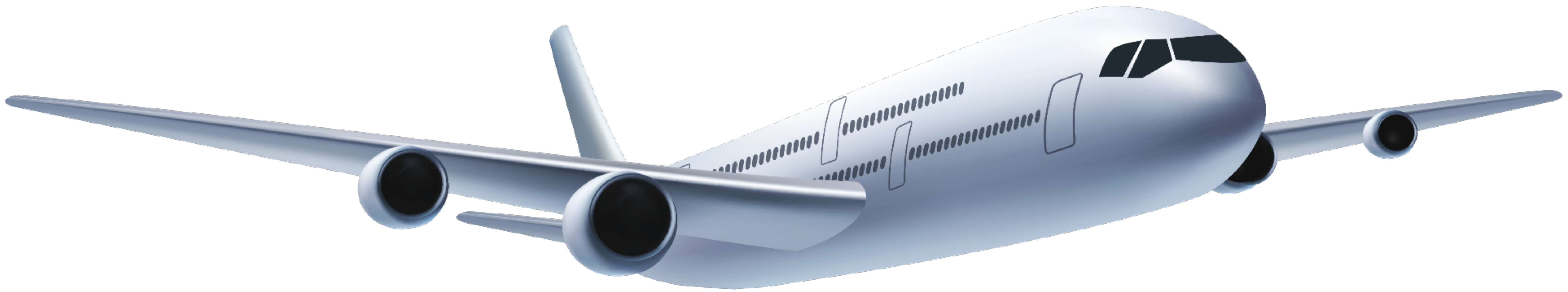
IMU



IMU

- Data related to orientation, velocity, and gravity





IMU

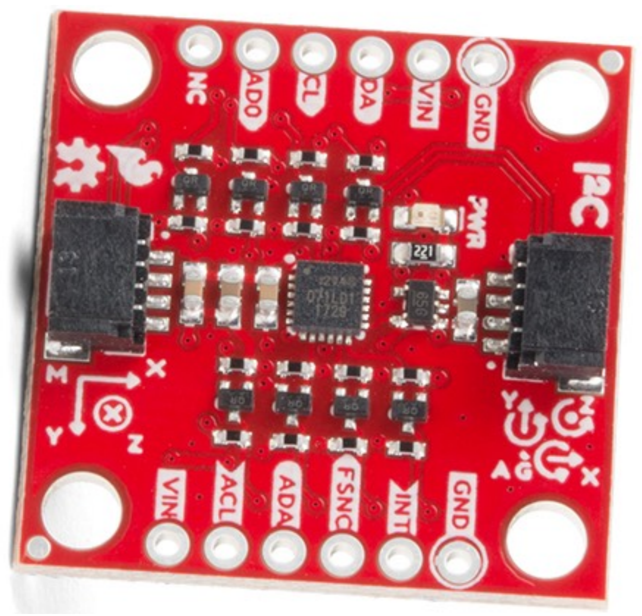
- Accelerometer
  - Linear acceleration, $a = \dot{v}$ [m/s^2]
- Gyroscope
  - Angular velocity, $\omega = \dfrac{\Delta\theta}{\Delta t}$ [°/s]
- Magnetometer
  - Magnetic field strength, [uT] or [Gauss]
- NB: Gravity, magnetic fields, accelerations affect these sensors in many ways!

▶ Track orientation (position)

▶ Track orientation

Dead reckoning

▶ Get absolute orientation

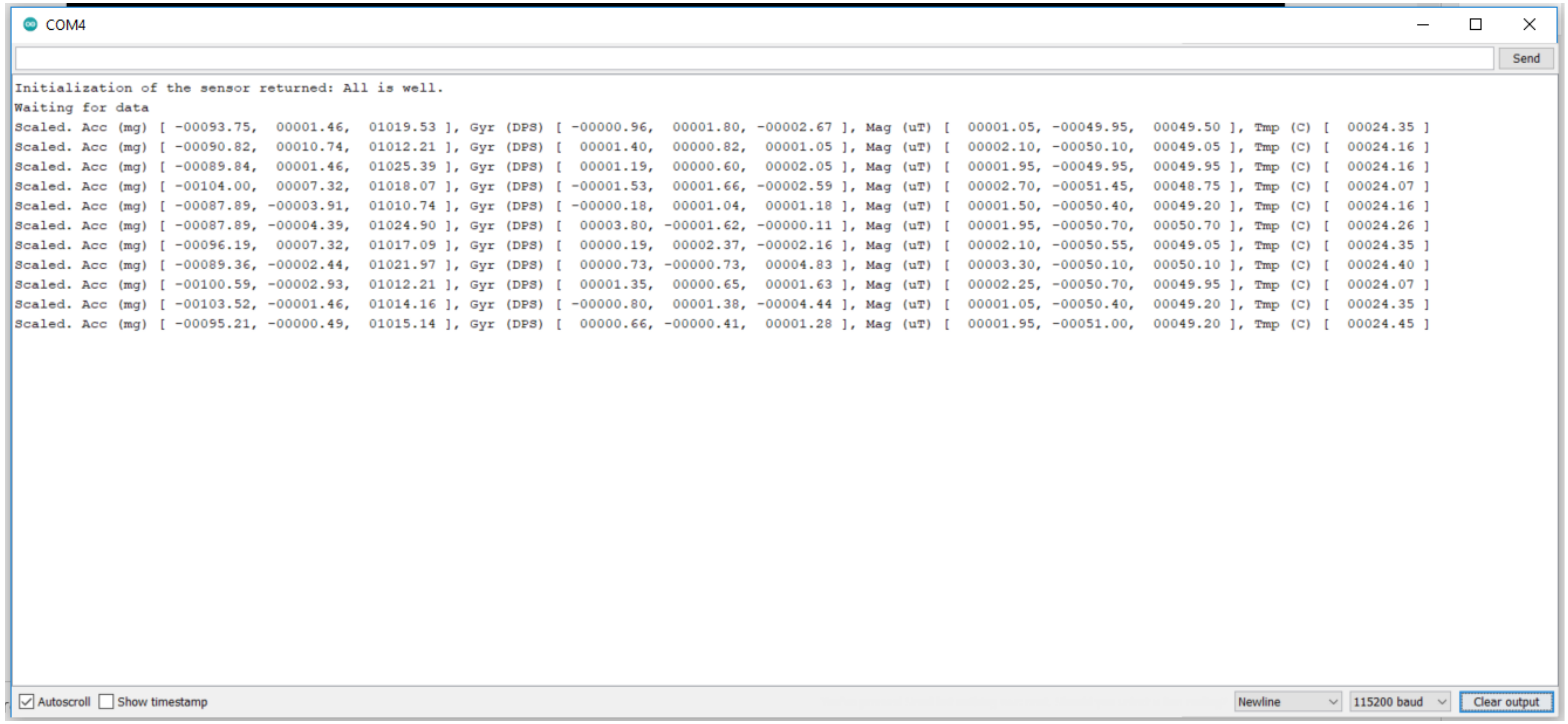


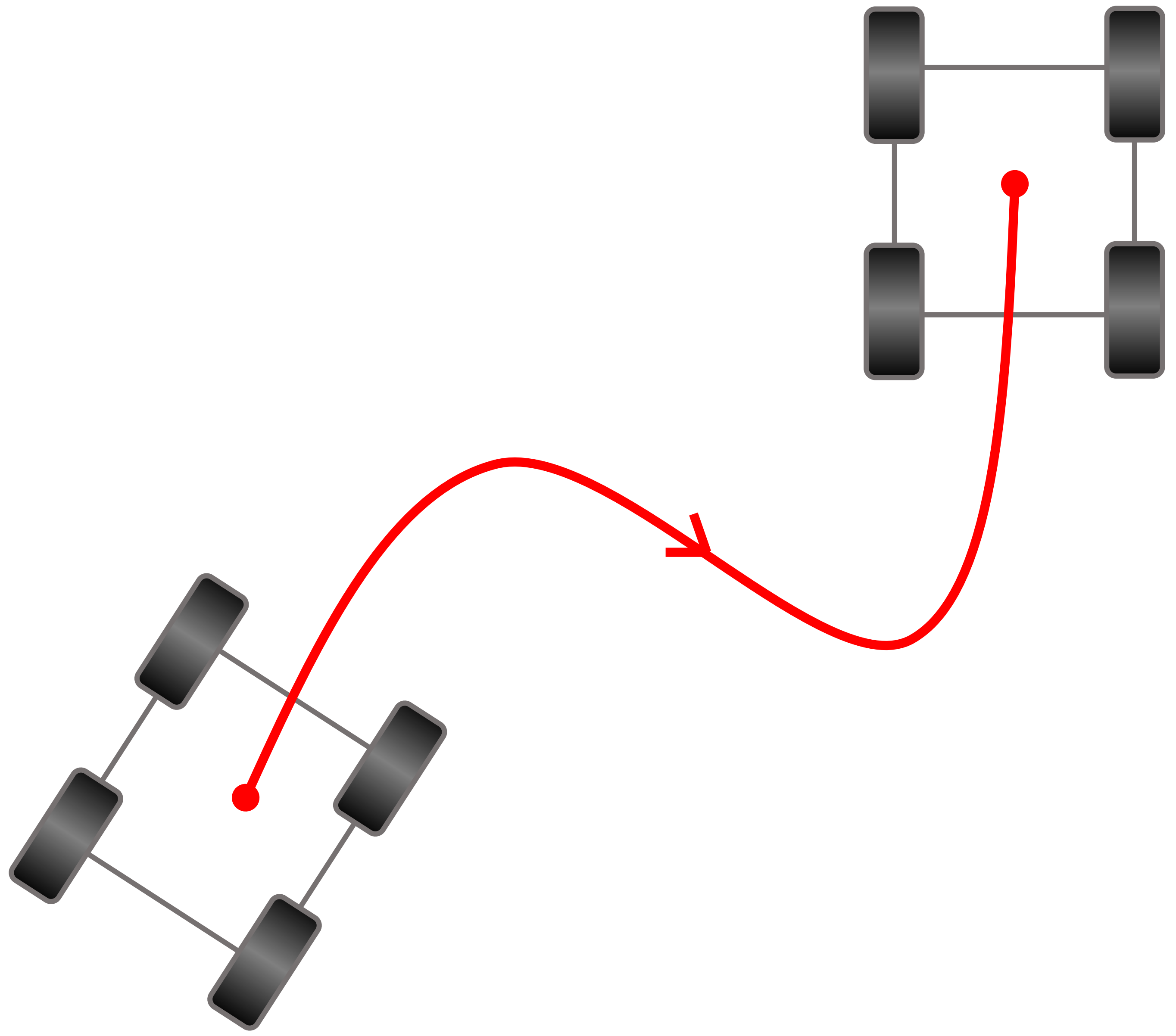
ICM-20948
- $16
- Low power
- 9-axis

IMU

Demo

- Install Sparkfun 9DOF IMU — ICM 20948 library
- Follow the basics example





Accelerometer

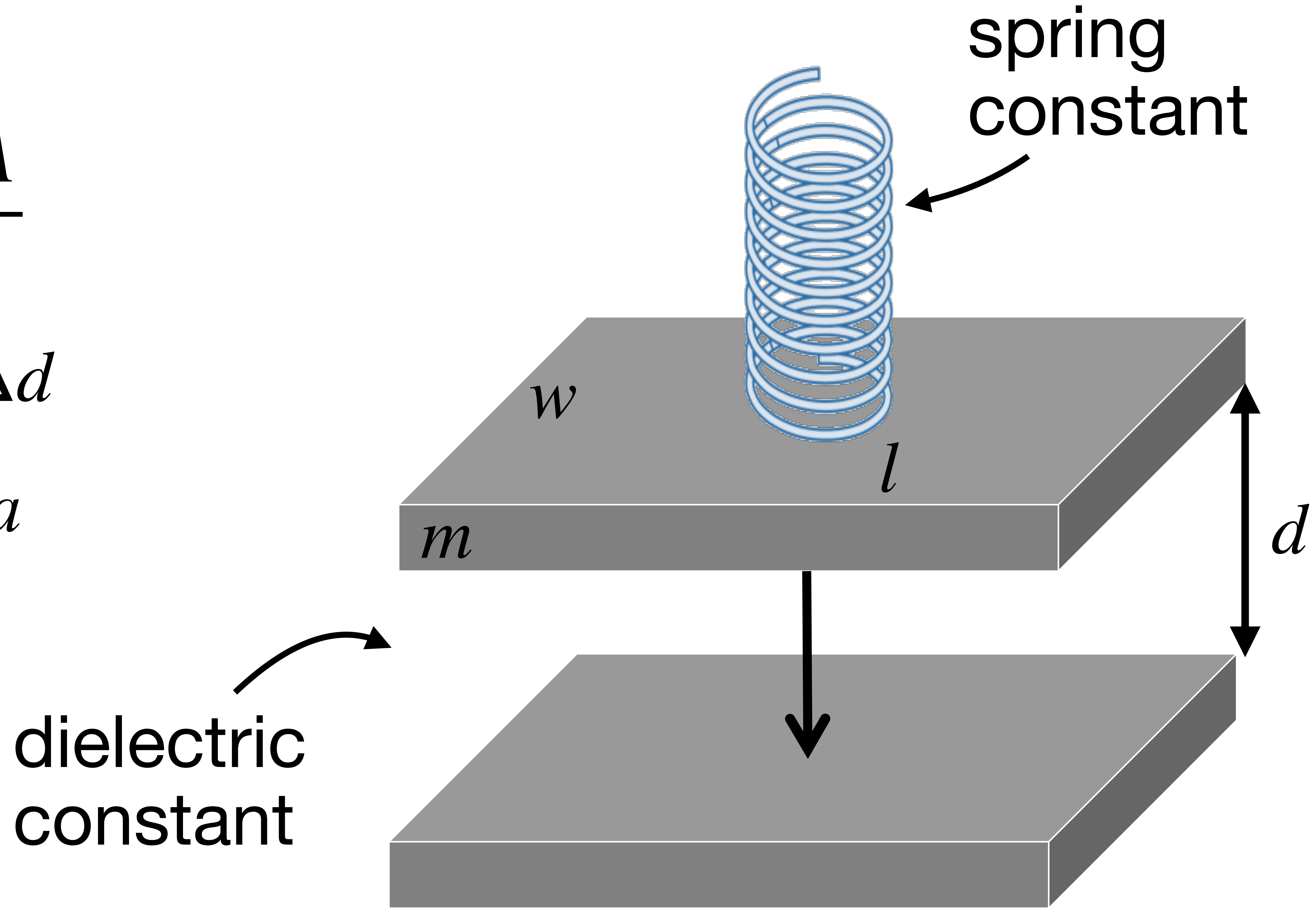
Accelerometer

Measure acceleration

$$C = \frac{\epsilon A}{d}$$

$$F = k\Delta d$$

$$F = ma$$



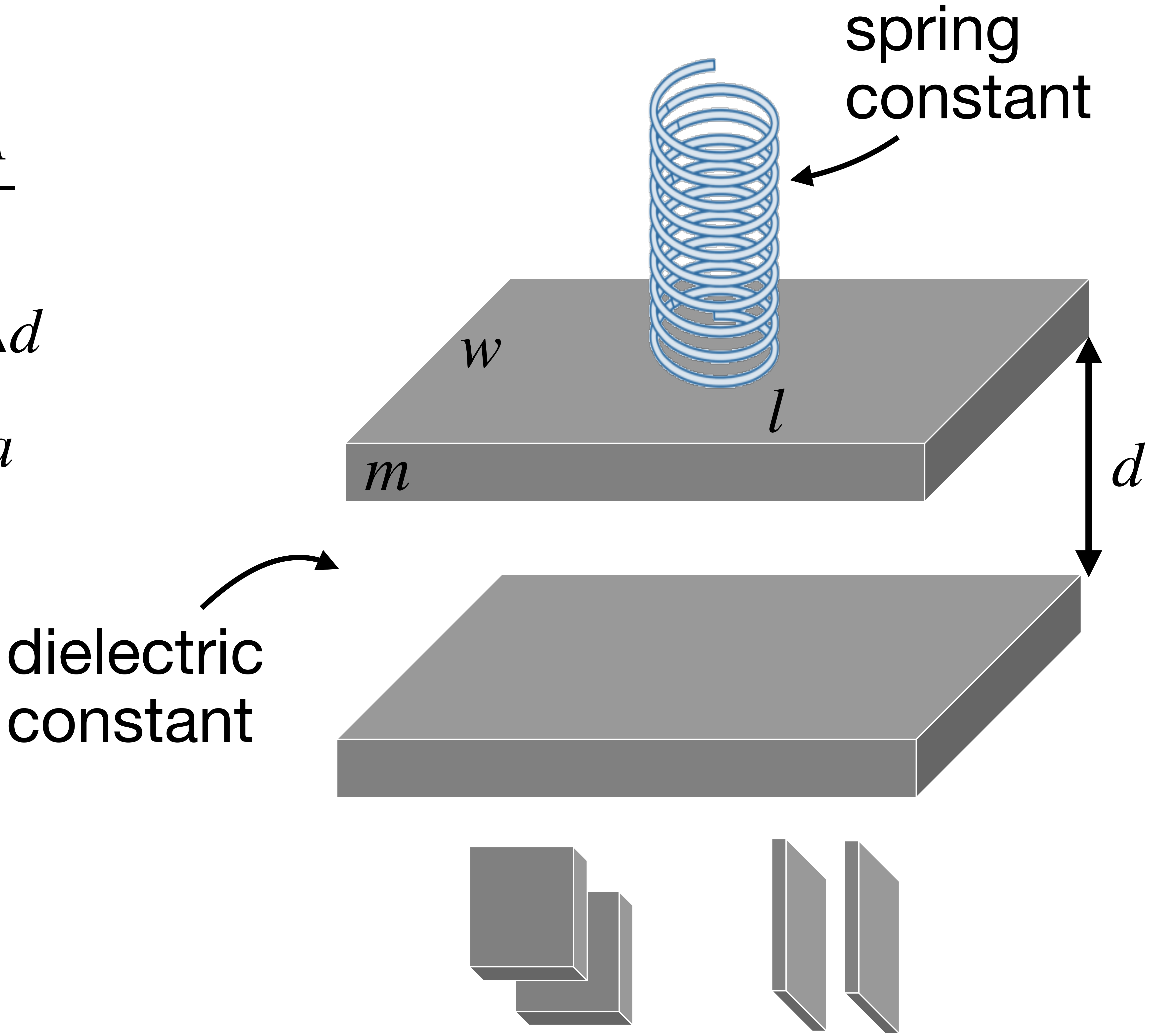
Accelerometer

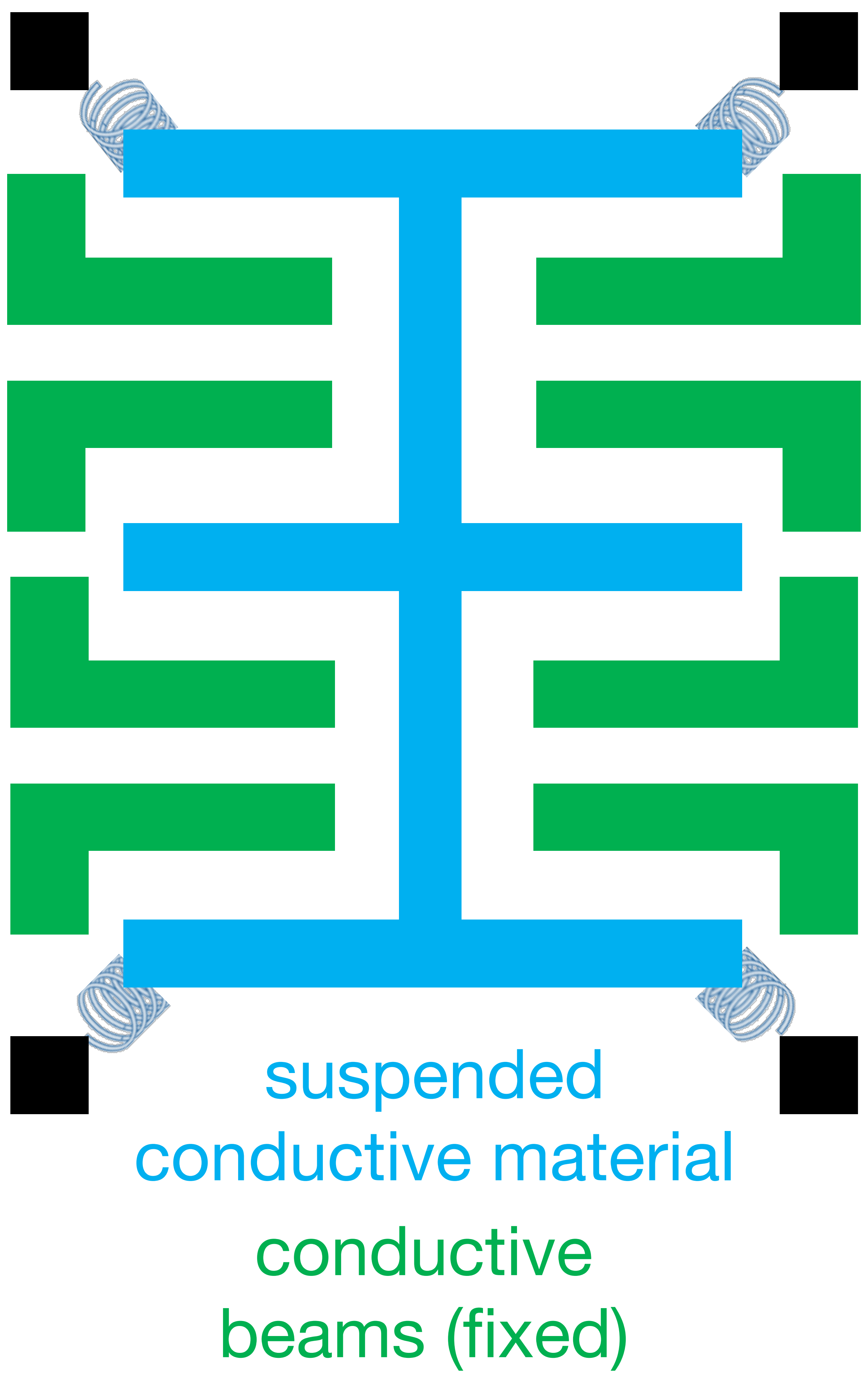
Measure acceleration in 3D

$$C = \frac{\epsilon A}{d}$$

$$F = k\Delta d$$

$$F = ma$$

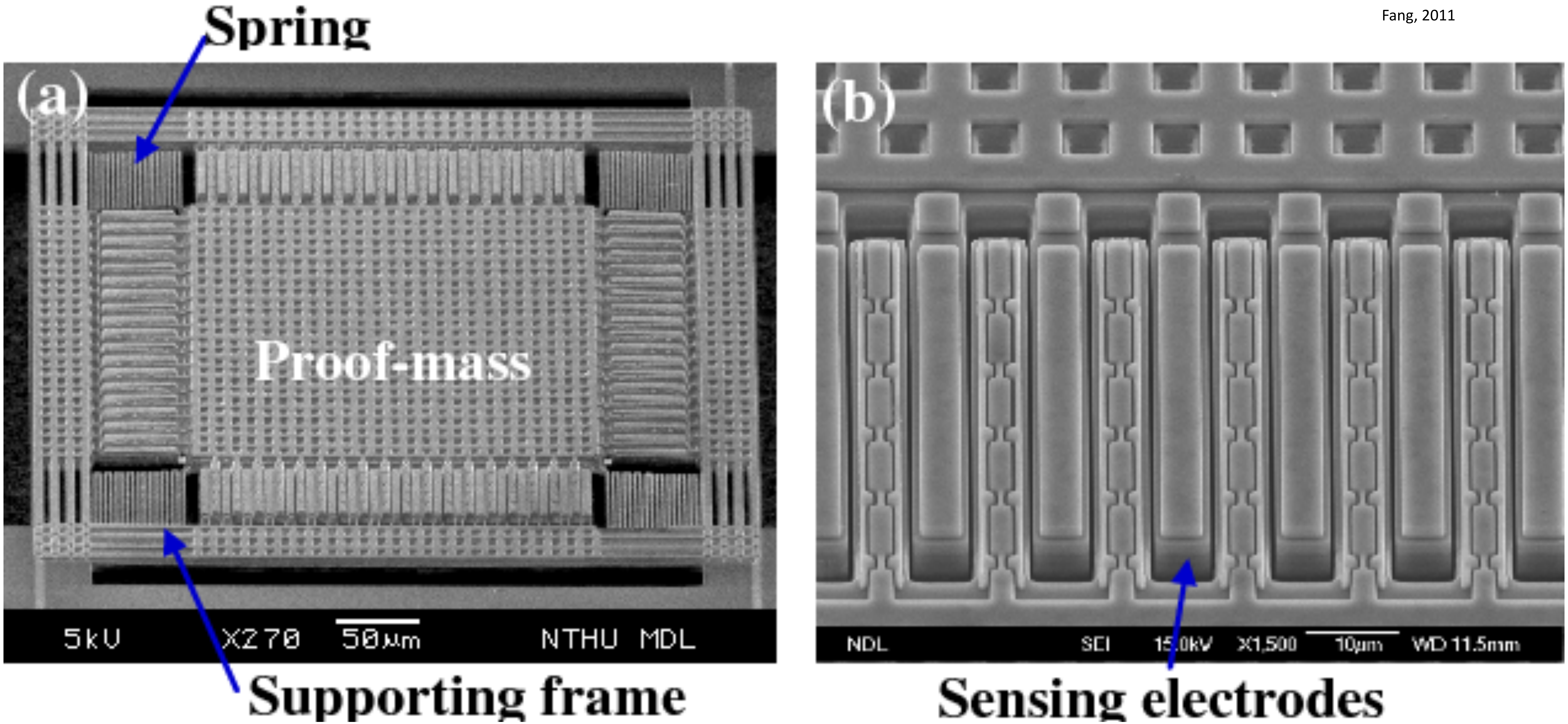




Accelerometer

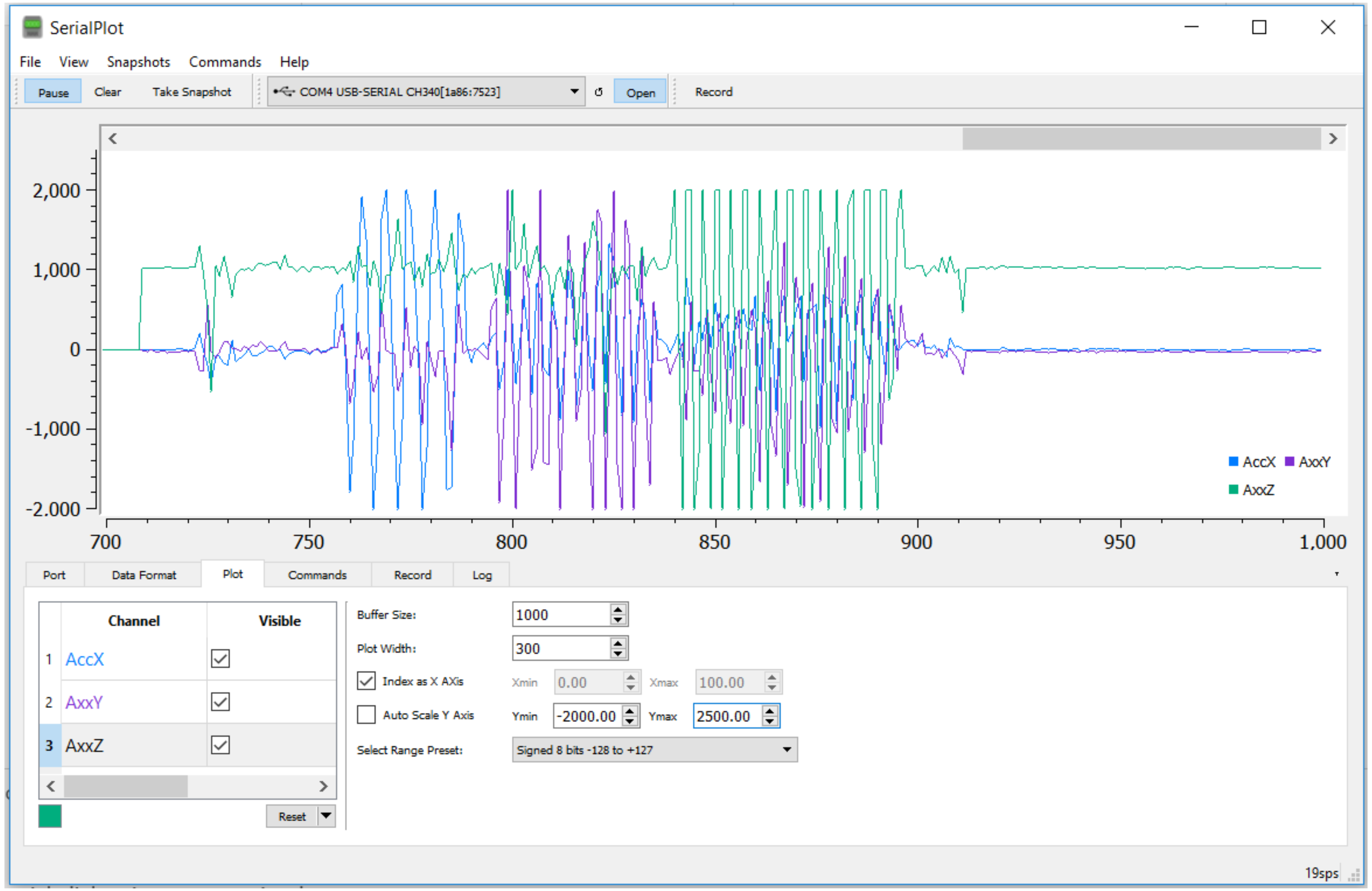
Measure acceleration in 3D

Fang, 2011



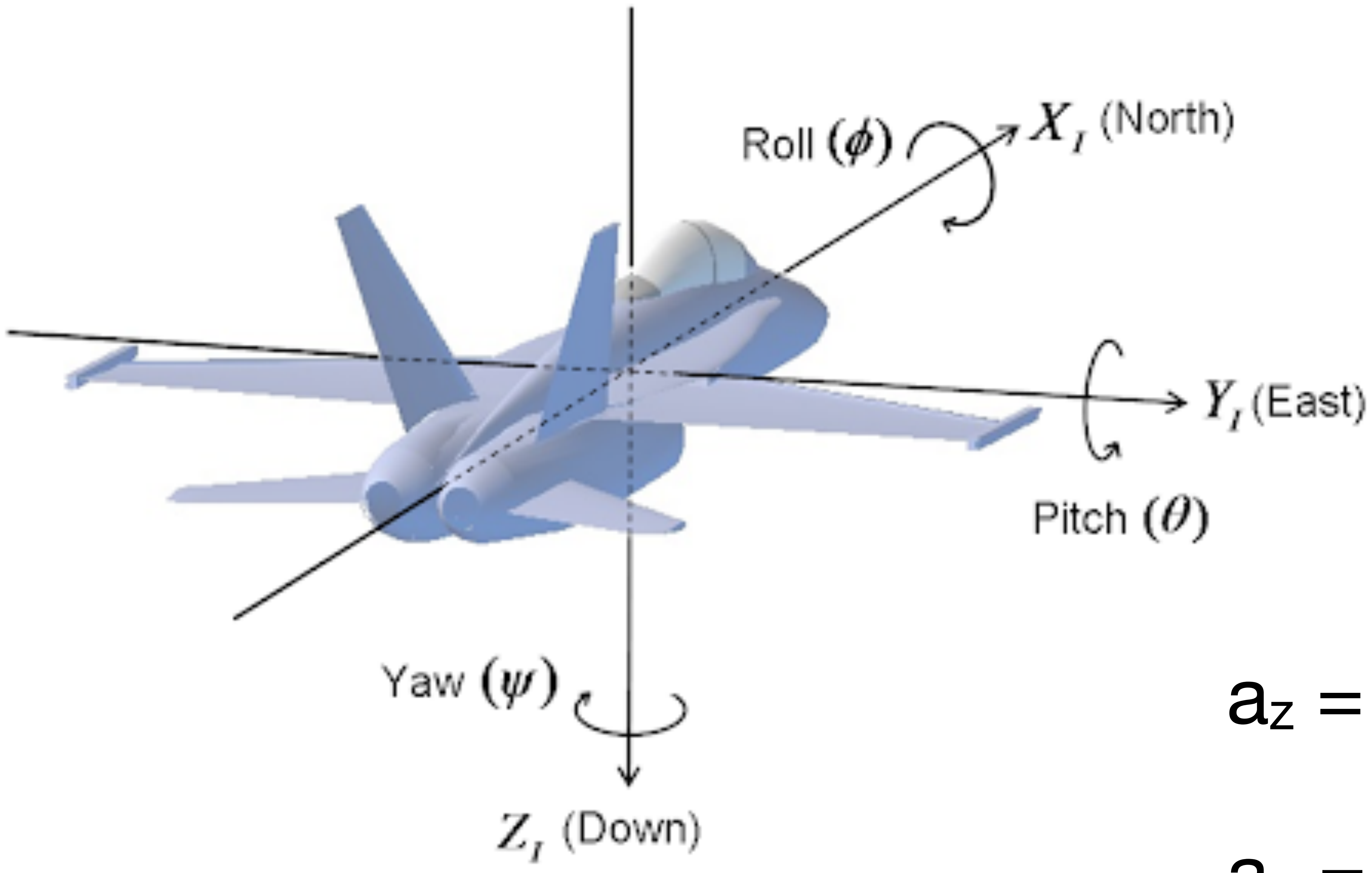
Accelerometer

Use Serial Monitor or Serial Plot



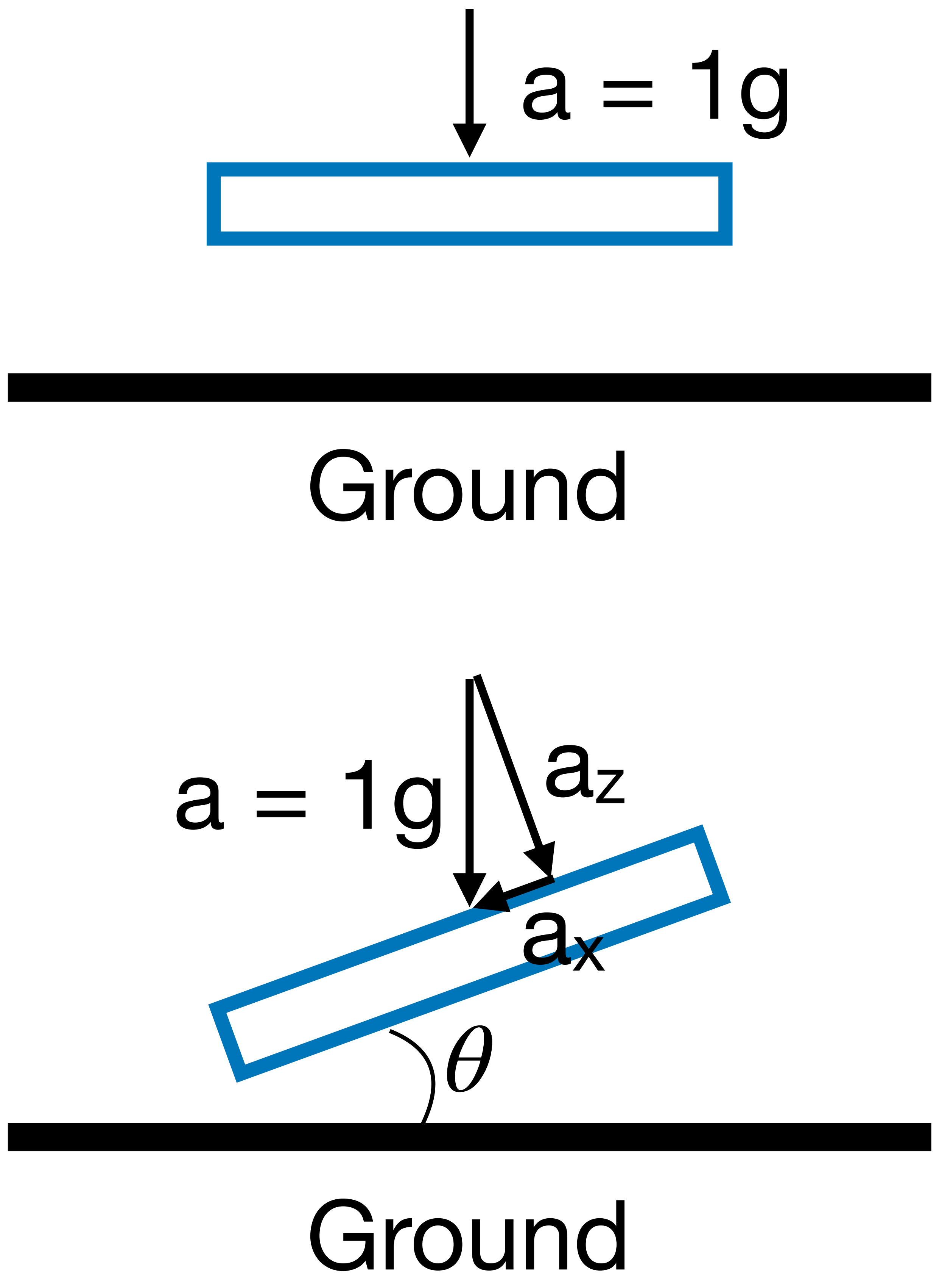
Accelerometer

Roll, Pitch, Yaw



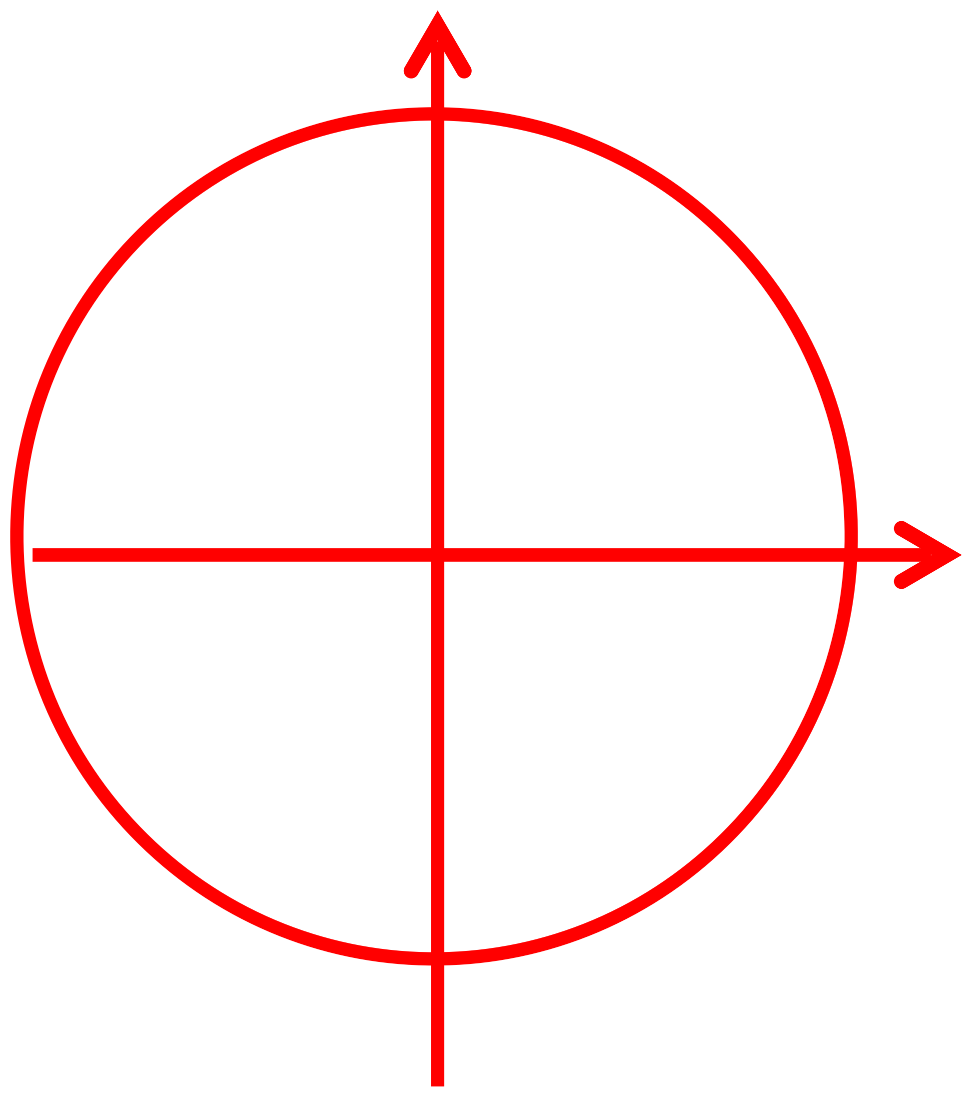
$$a_z = 1g\cos(\theta)$$

$$a_x = 1g\sin(\theta)$$



atan vs. atan2

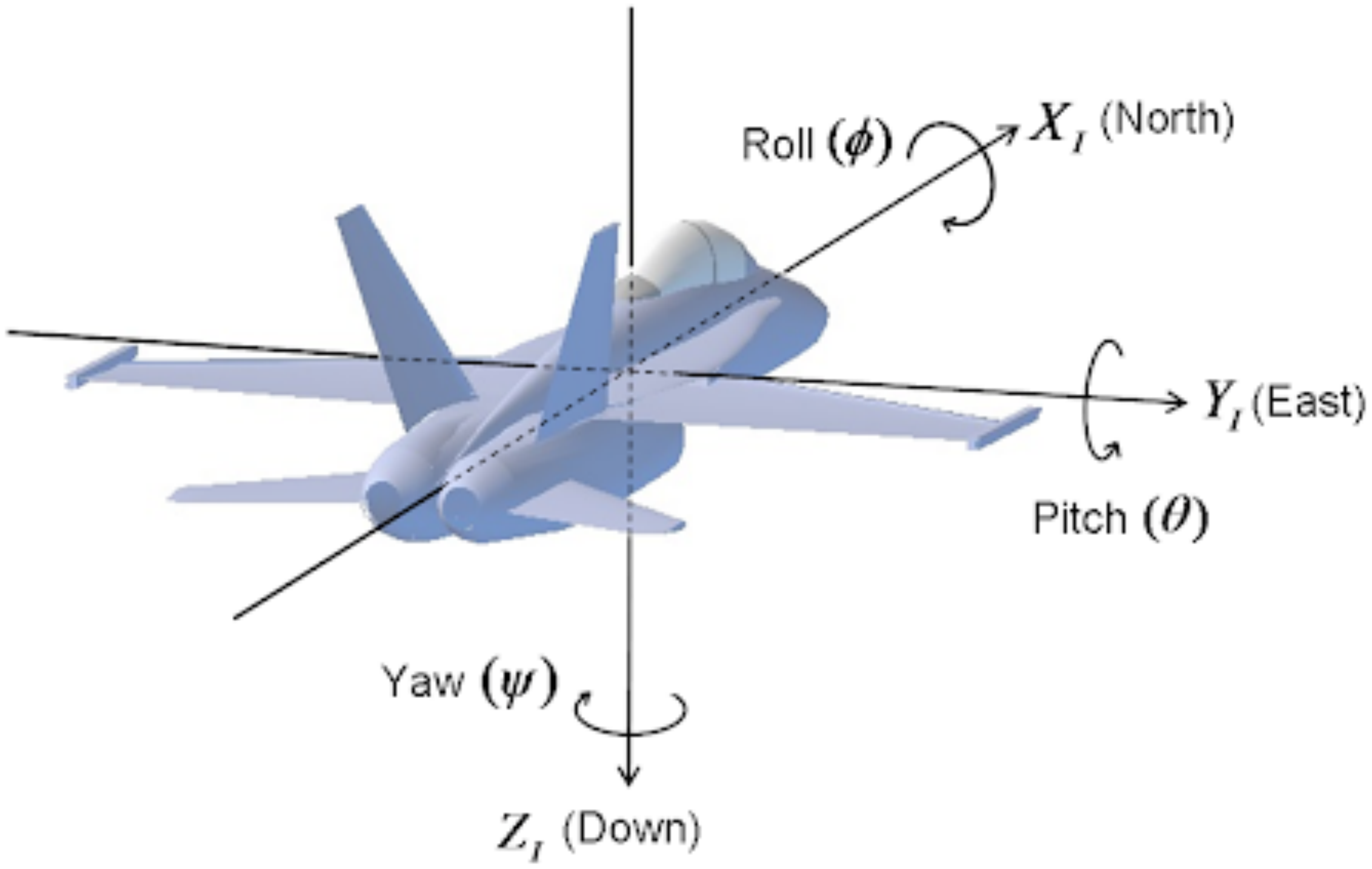
- atan(a_x ,a_z) returns [-π/2, π/2]
- Instead use atan2(a_x ,a_z) which returns [-π, π]

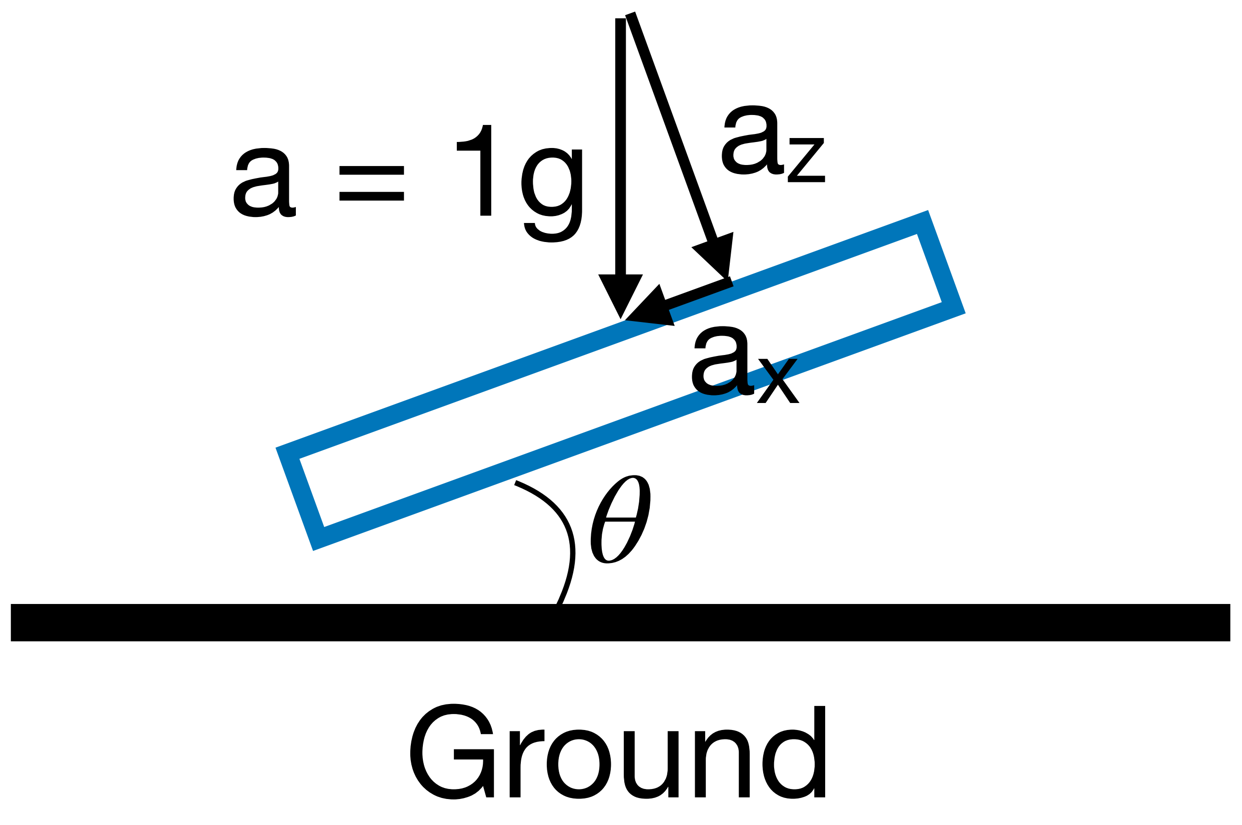


```
float atan2(float x, float y) {
    if (x > 0.0)
        return atan(y/x);
    if (x < 0.0) {
        if (y >= 0.0)
          return (PI + atan(y/x));
        else
          return (-PI + atan(y/x));
        }
    if (y > 0.0) // x == 0
        return PI_ON_TWO;
    if (y < 0.0)
        return -PI_ON_TWO;
    return 0.0; // Should be undefined
}
```

Accelerometer

Roll, Pitch, ~~Yaw~~





$$a_z = 1g\cos(\theta)$$

$$a_x = 1g\sin(\theta)$$

$$\theta = \mathrm{atan2}(a_x, a_z)$$

$$\phi = \mathrm{atan2}(a_y, a_z)$$

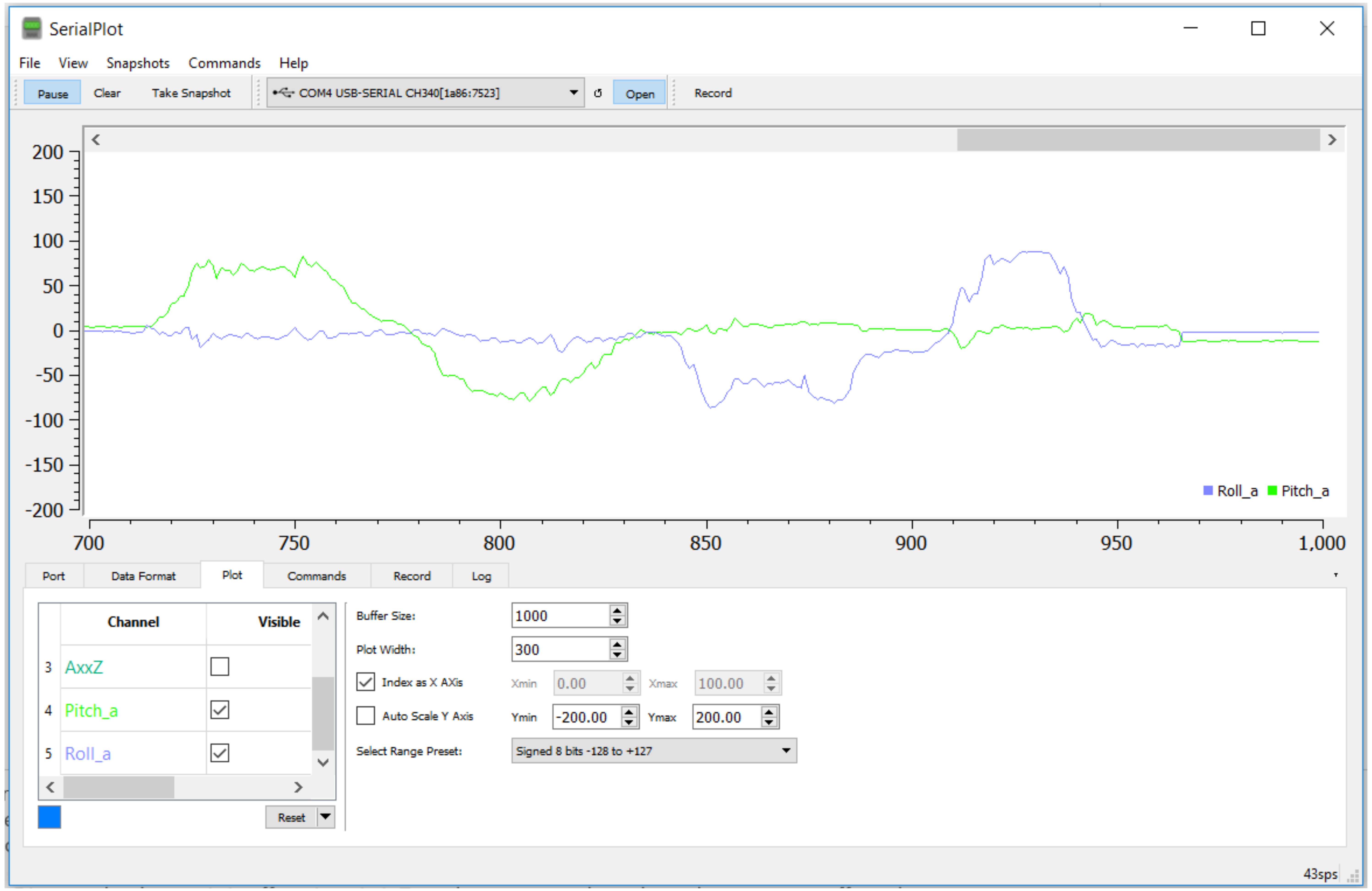
Can we estimate yaw?

Accelerometer

Pitch and Roll

$$\theta = \text{atan2}(a_x, a_z)$$

$$\phi = \text{atan2}(a_y, a_z)$$



Accelerometer

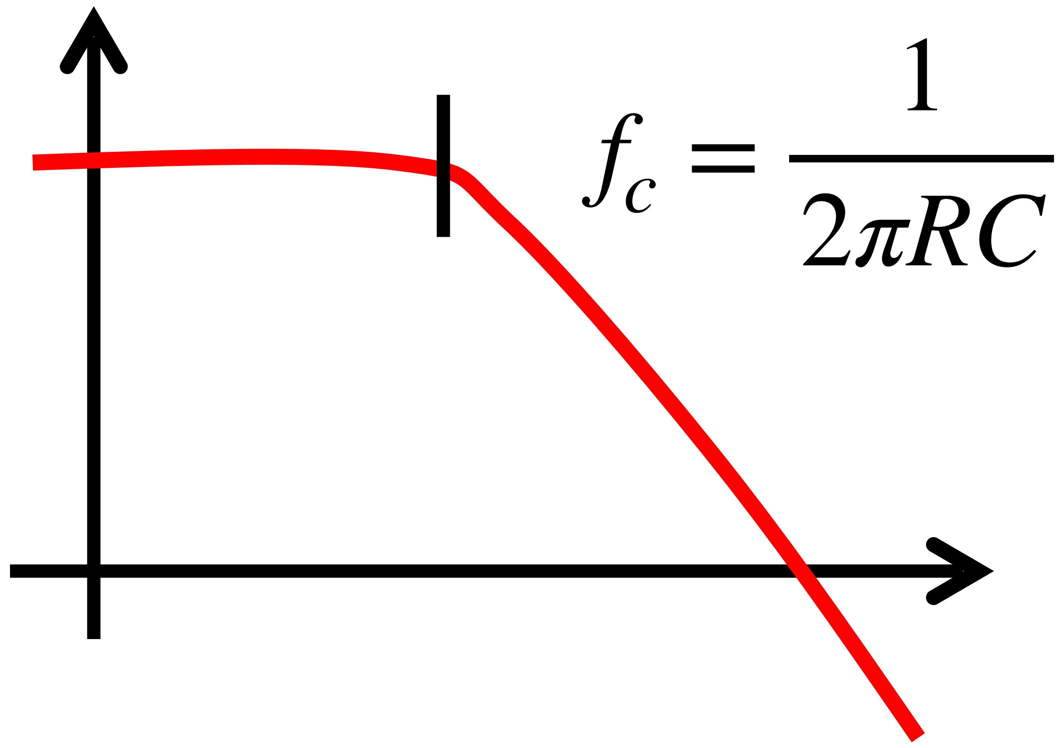
Roll and Pitch

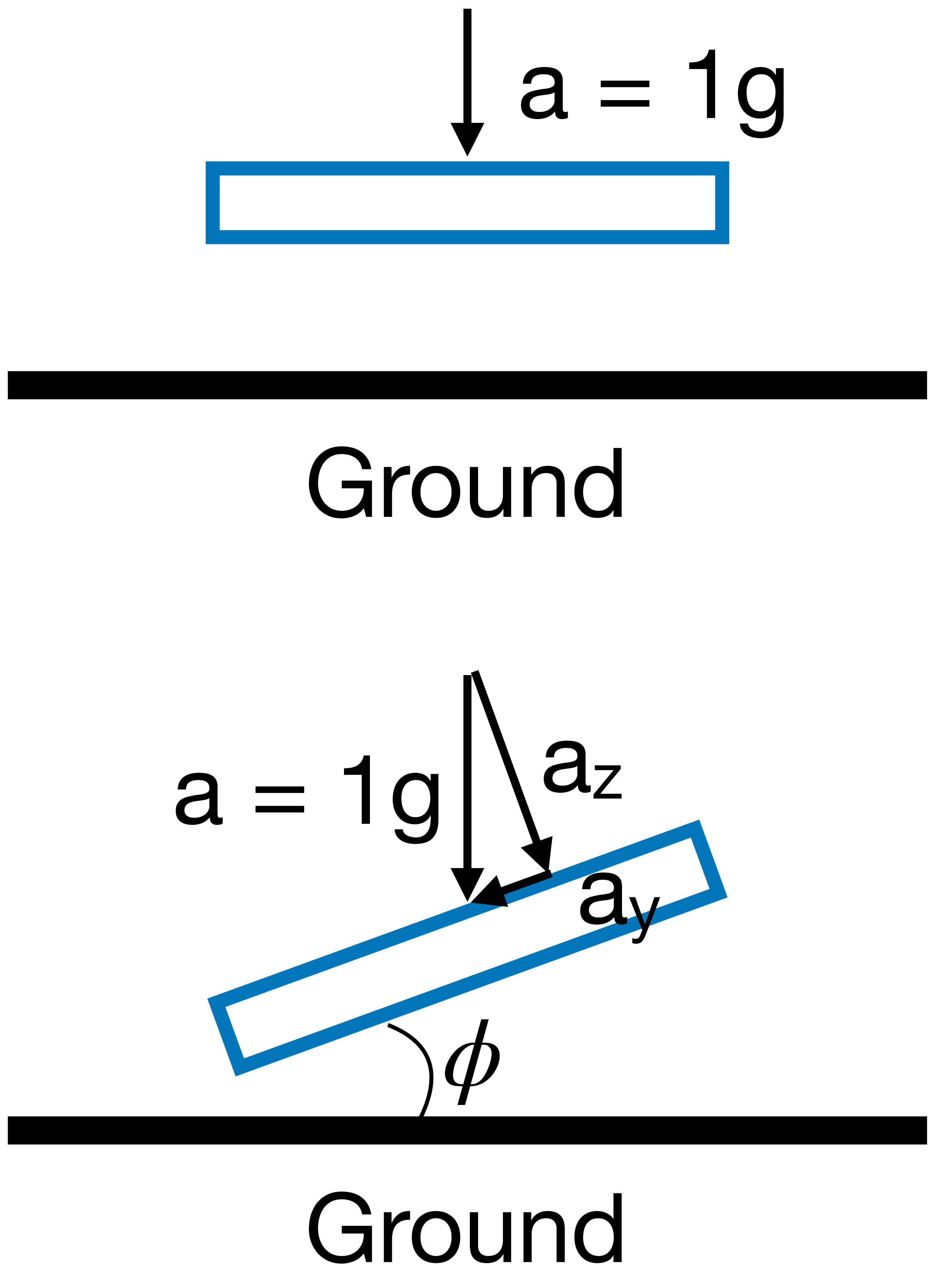
- Good (very accurate on average) vs. bad (noisy)
- Low pass filter

$$\theta_{LPF}[n] = \alpha * \theta_{RAW} + (1 - \alpha) * \theta_{LPF}[n-1]$$
$$\theta_{LPF}[n-1] = \theta_{LPF}[n]$$

- Think of your frequency like an RC low-pass filter:

$$\alpha = \frac{T}{T + RC}$$





Accelerometer

Dead Reckoning

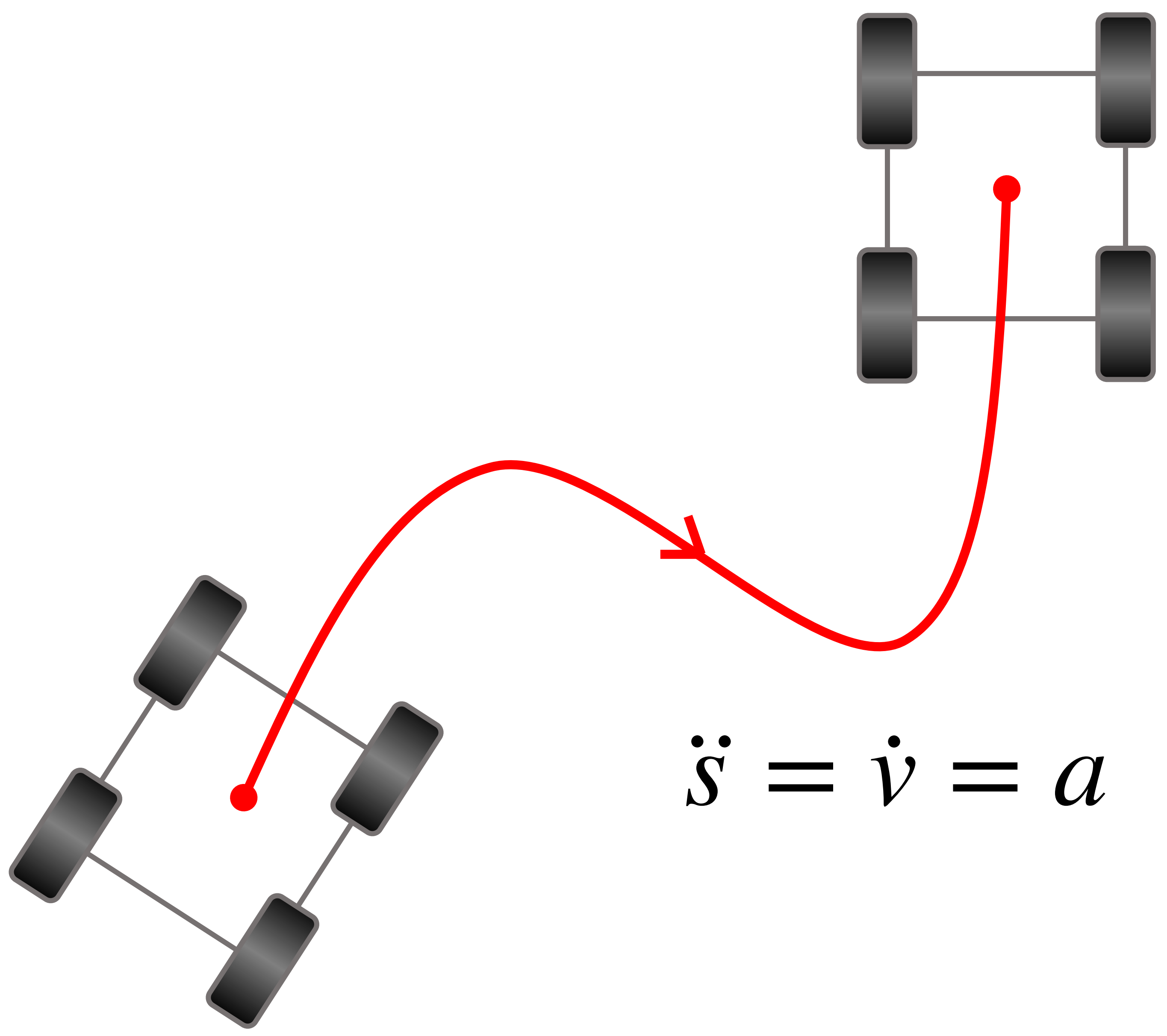
- Use the accelerometer to do dead reckoning?

$$v = \int a$$

$$s = \int\int a$$

```
v[k+1] = v[k] + a[k]*dt
s[k+1] = s[k] + v[k]*dt
```

- If you do this at home, remember unit conversion! Accelerometer output is in mg (1g ~9.81 m/s²)



$$\ddot{s} = \dot{v} = a$$

Accelerometer

Dead Reckoning

- Issue: Distinguishing sensor acceleration from gravity
  - **Solution 1**: Calibrate the offset
  - **Solution 2**: Low pass filter
  - **Solution 3**: Minimum signal cut-off

Errors only accumulate, and they grow fast!

www.chrobotics.com

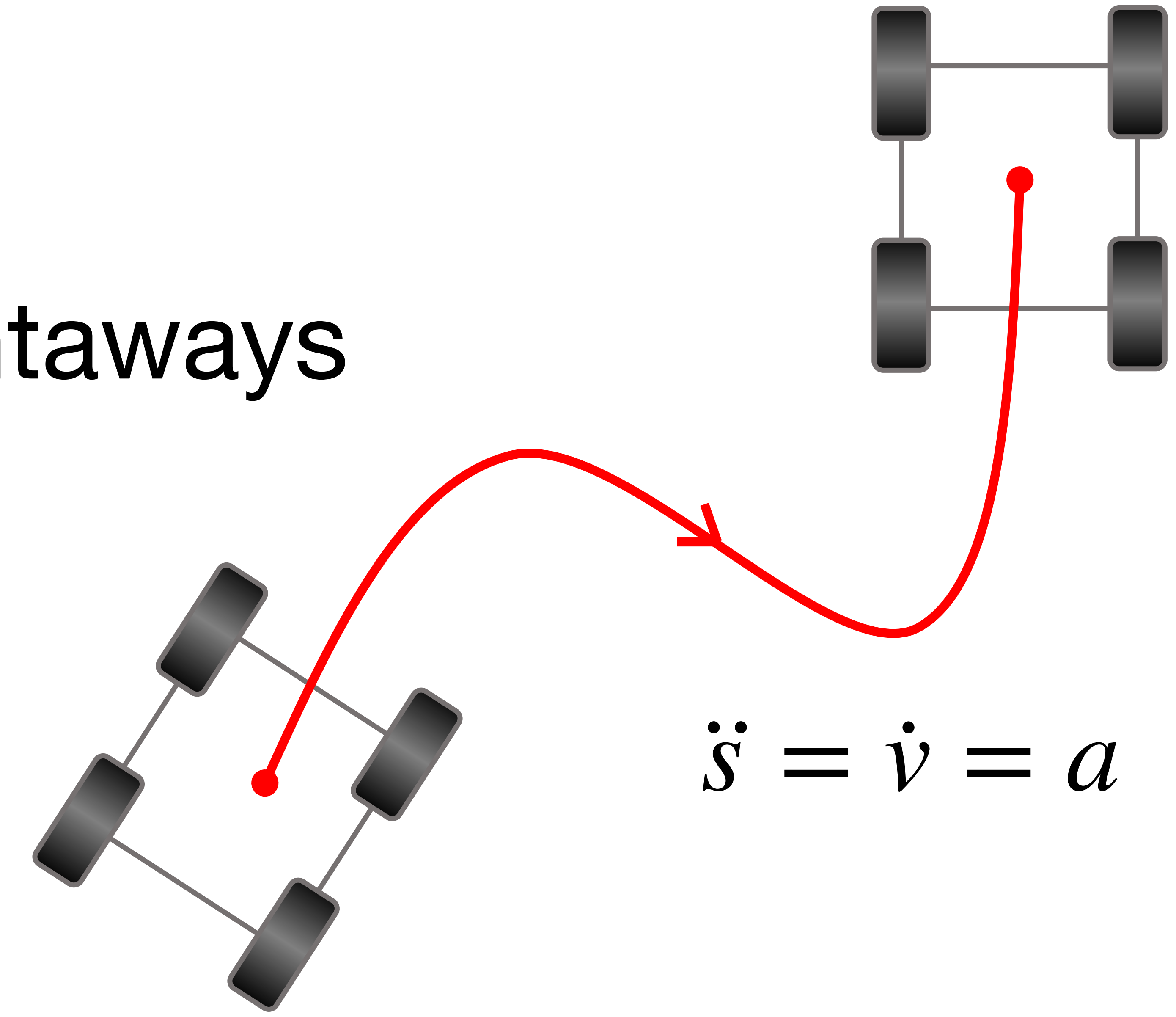
Angle Error (degrees)	Acceleration Error (m/s/s)	Velocity Error (m/s) at 10 seconds	Position Error (m) at 10 seconds	Position Error (m) at 1 minute	Position Error (m) at 10 minutes	Position Error (m) at 1 hour
0.1	0.017	0.17	1.7	61.2	6120	220 e 3
0.5	0.086	0.86	8.6	309.6	30960	1.1 e 6
1.0	0.17	1.7	17	612	61200	2.2 e 6
1.5	0.256	2.56	25.6	921.6	92160	3.3 e 6
2.0	0.342	3.42	34.2	1231.2	123120	4.4 e 6
3.0	0.513	5.13	51.3	1846.8	184680	6.6 e 6
5.0	0.854	8.54	85.4	3074.4	307440	11 e 6

Table 1 - A summary of velocity and position errors caused by attitude estimation error.

Accelerometer

Dead Reckoning

- Issue: Distinguishing sensor acceleration from gravity
  - **Solution 1**: Calibrate the offset
  - **Solution 2**: Low pass filter
  - **Solution 3**: Minimum signal cut-off
  - **Solution 4**: Stop periodically and zero the velocity
  - **Solution 5**: Use in combination with ToF on straightaways
  - **Solution 6**: Buy a more expensive IMU
  - etc…



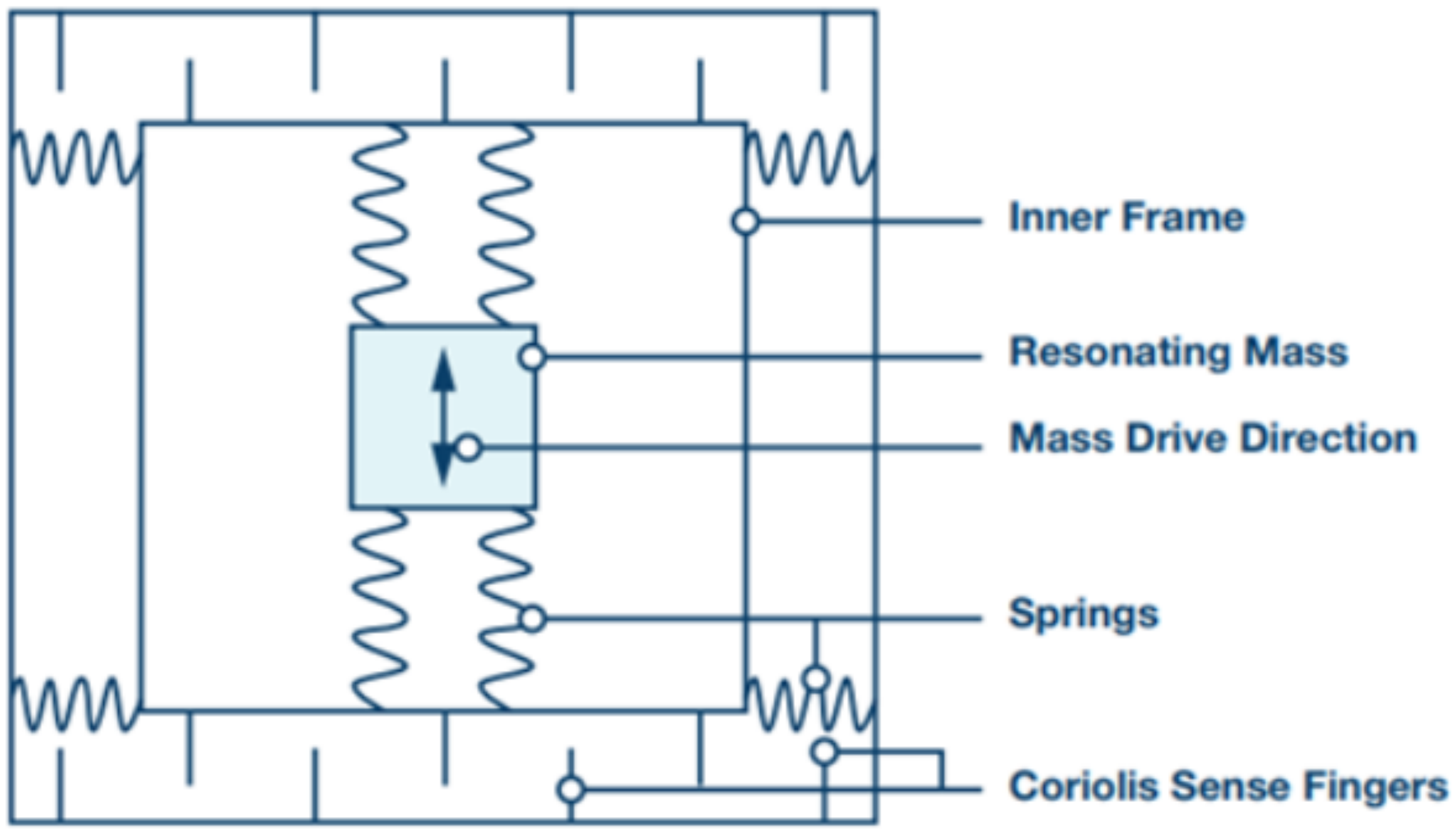
$$\ddot{s} = \dot{v} = a$$

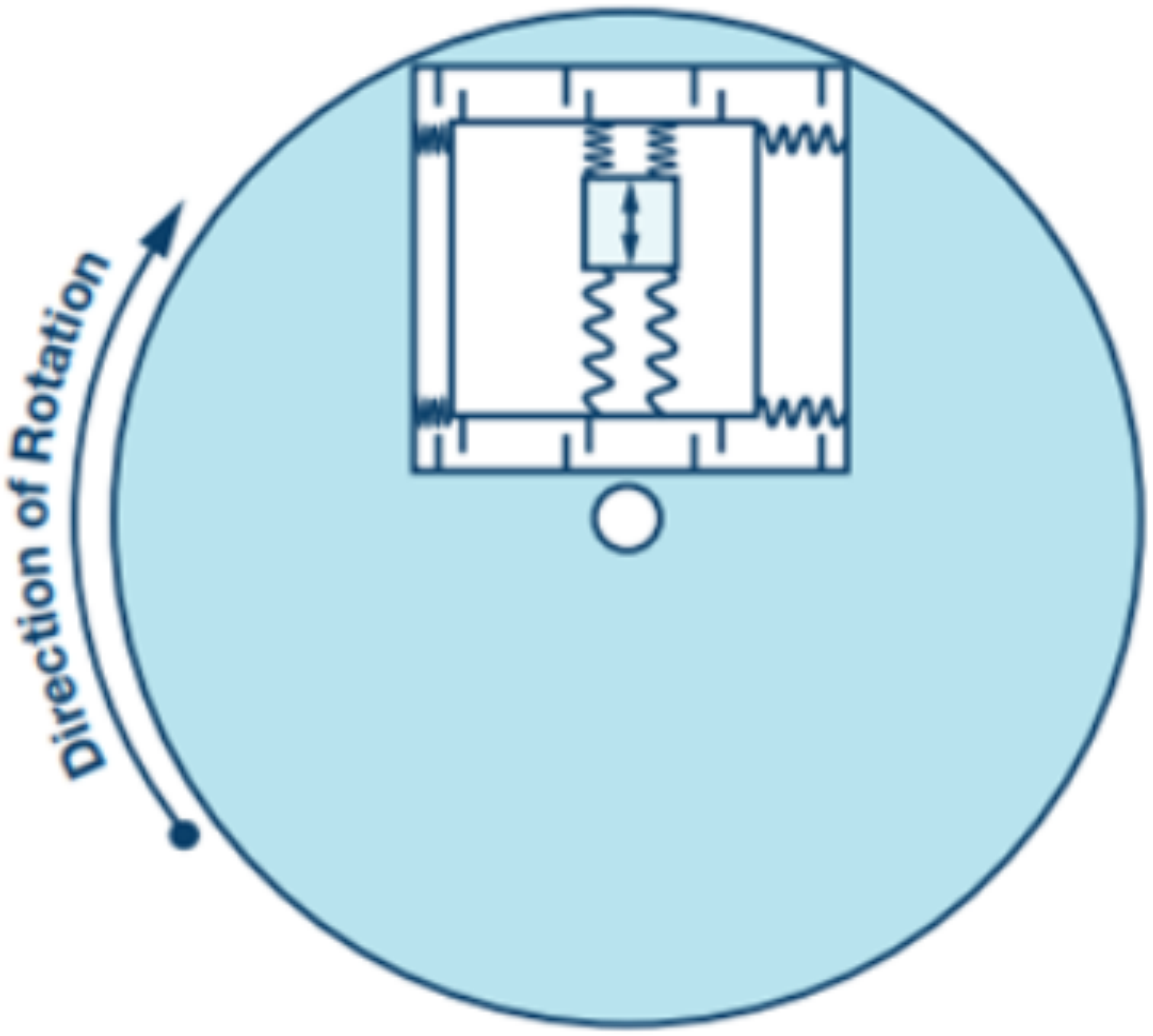
Gyroscope

Gyroscope

Measure rate of angular change [°/s]

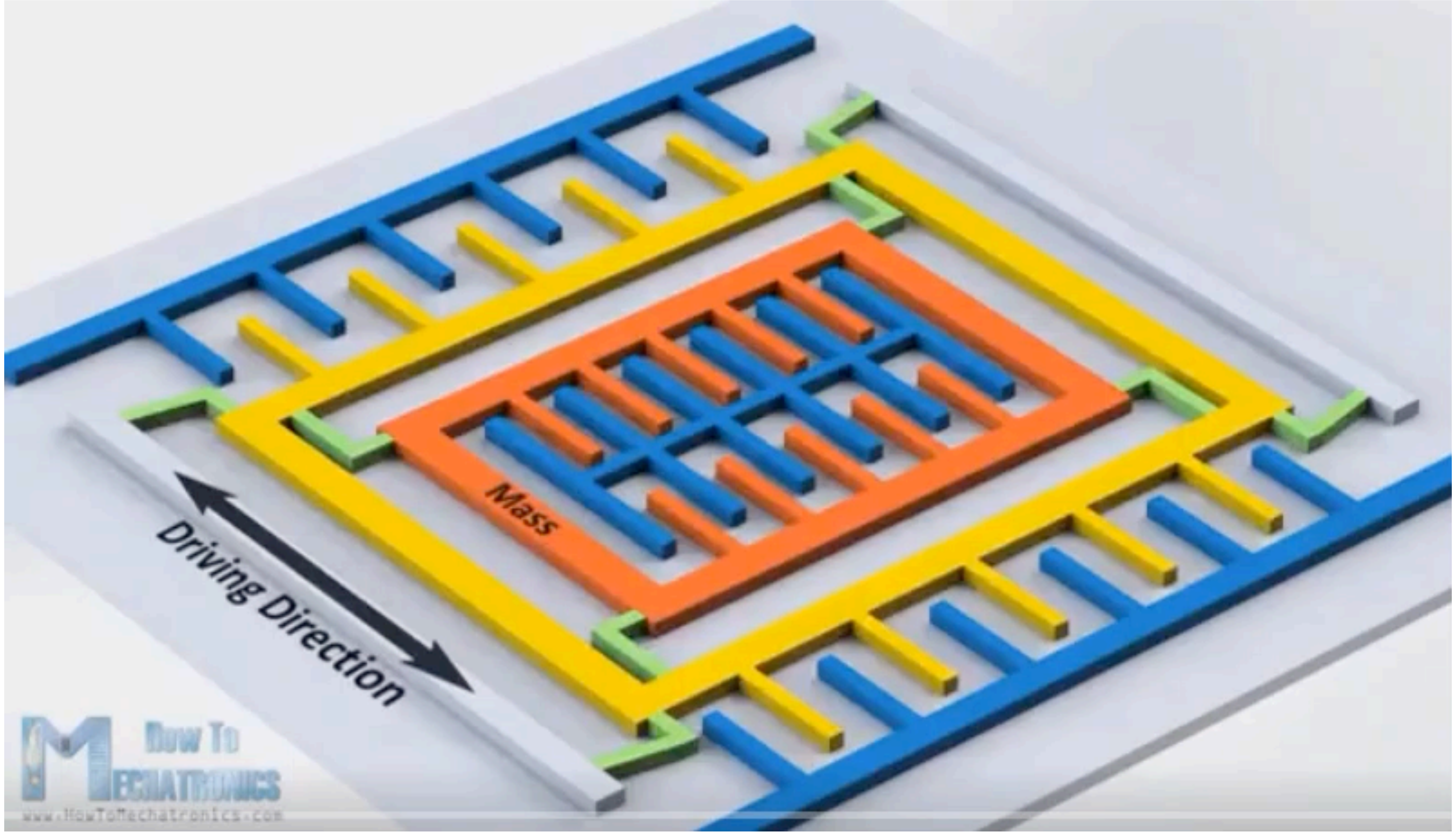
www.analog.com





Gyroscope

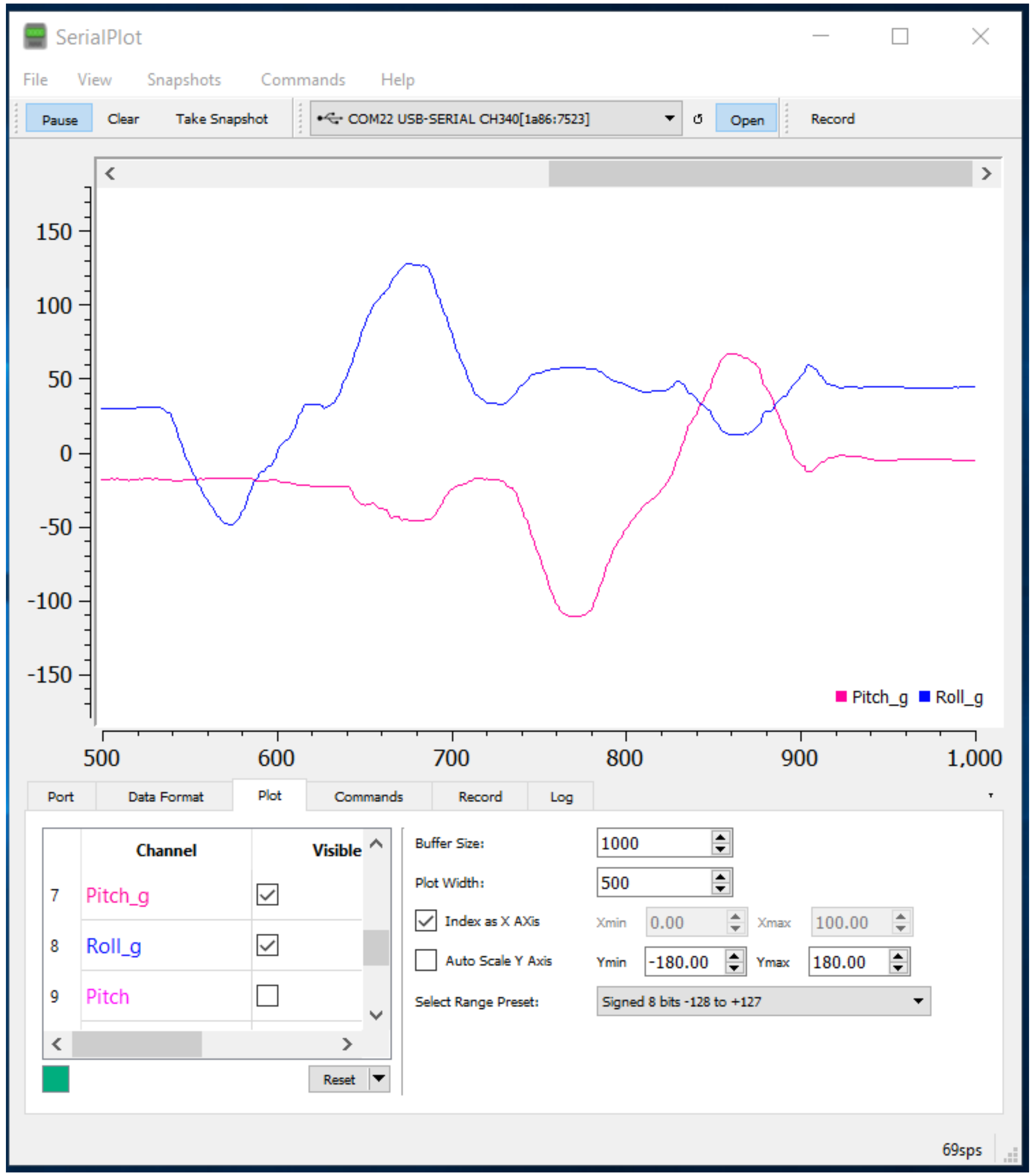
Measure rate of angular change [°/s]



Gyroscope

Measure rate of angular change [°/s]

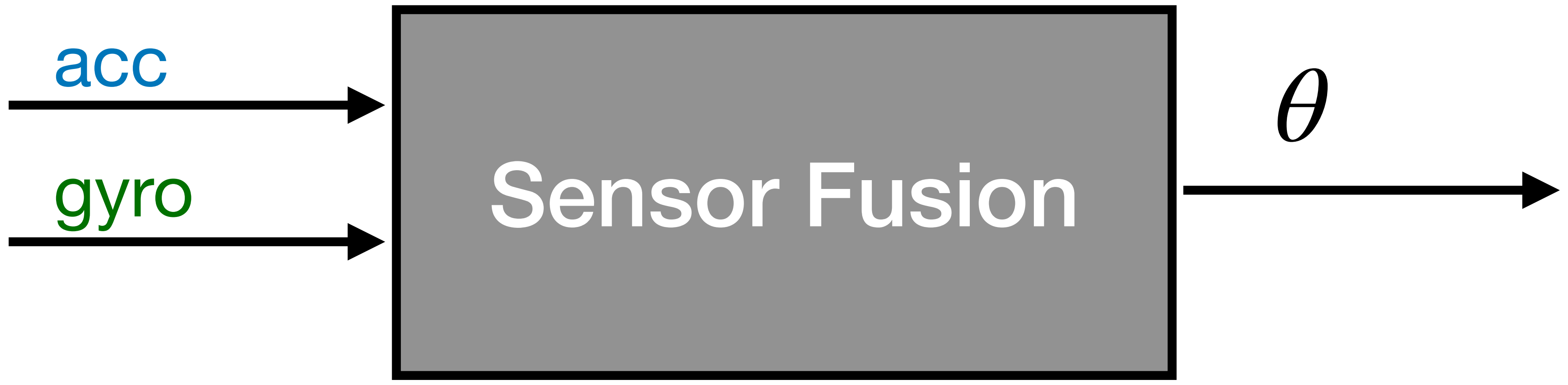
- How to measure angles?
  - $\theta_g = \theta_g + \mathrm{gyro\_reading} \times \boxed{\mathrm{dt}}$
- Drift, but low noise



Gyroscope

Measure rate of angular change [°/s]

- How to measure angles?
  - $\theta_g = \theta_g + \text{gyro\_reading} \times \text{dt}$
- Drift, but low noise
- Complementary filter:
  - $\theta = (\theta + \theta_g)(1 - \alpha) + \theta_a \alpha$



Can we estimate yaw?

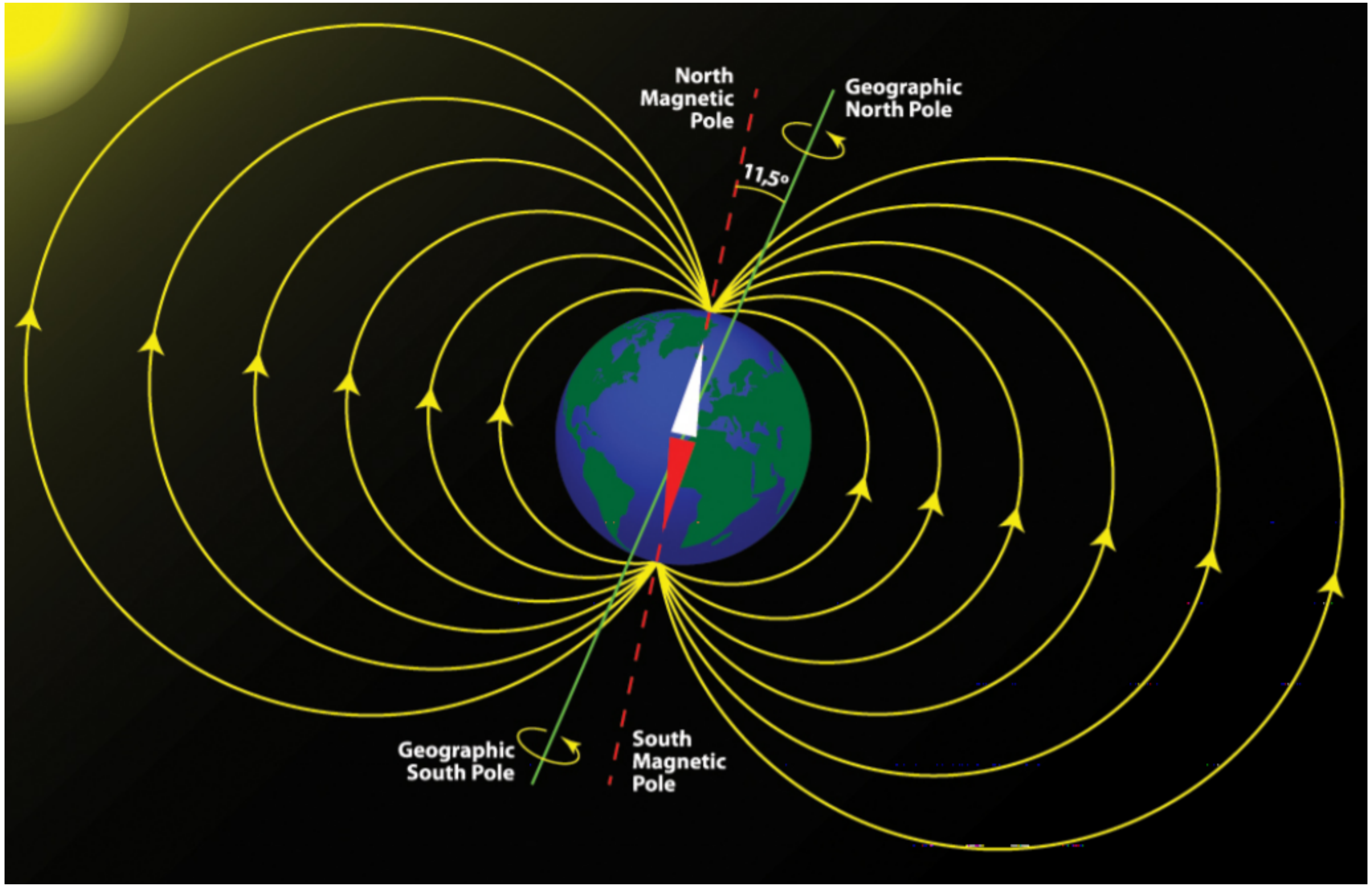
Yes, but no complementary data from the accelerometer

Magnetometer

Magnetometer

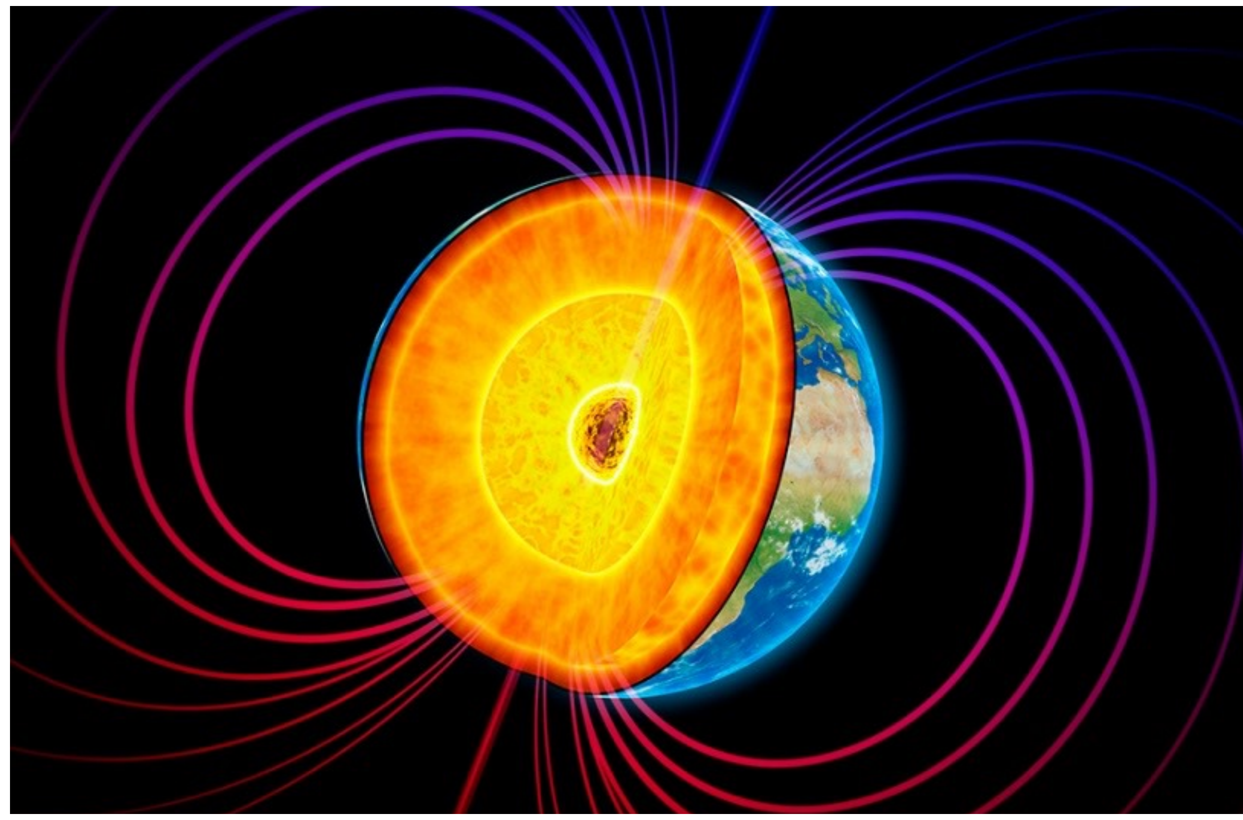
Measure Earth's magnetic field

- Depends on location, time



Magnetometer

Measure Earth's magnetic field



- Depends on location, time
- Distortion due to metal objects or nearby EM fields

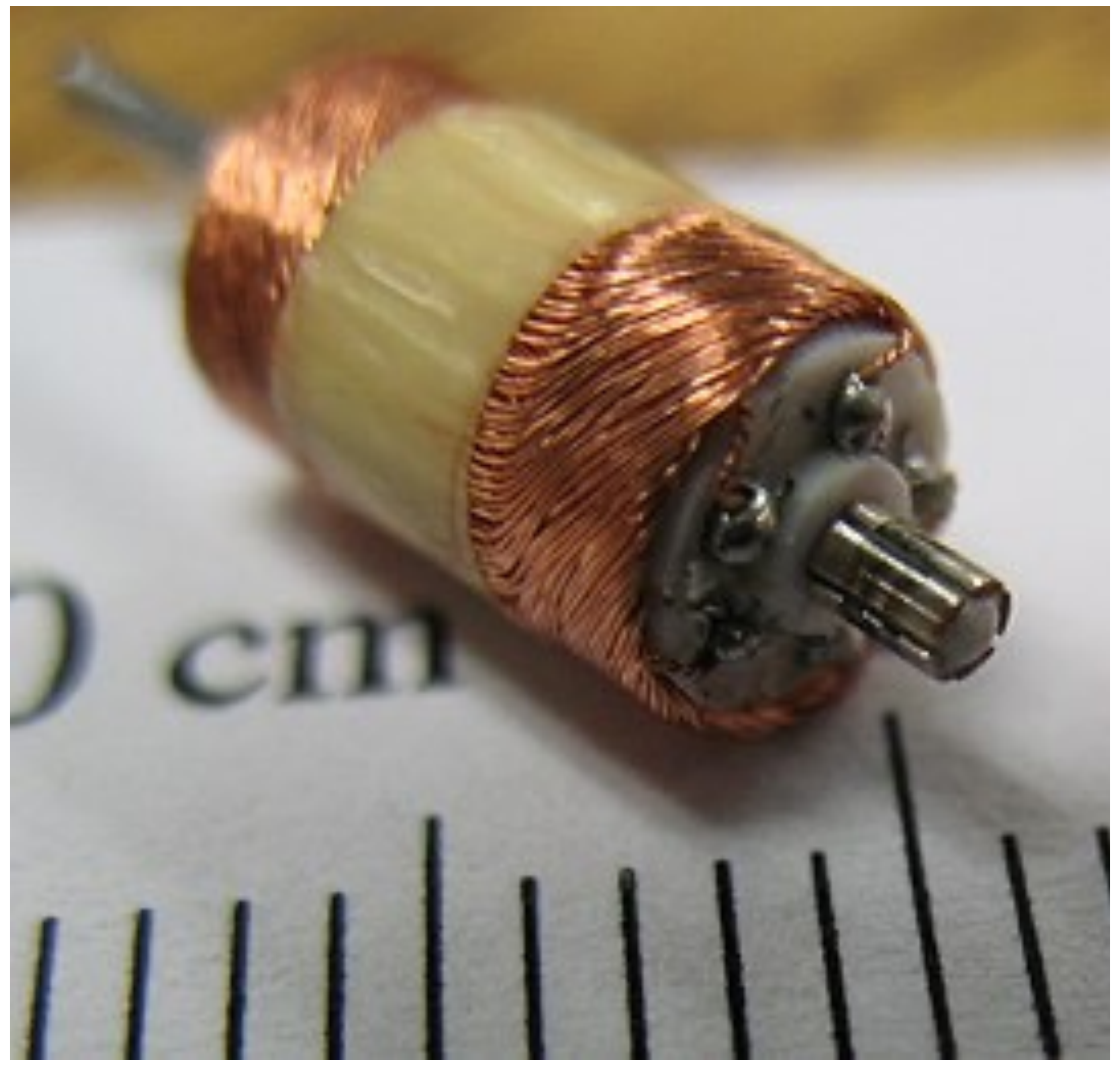
Magnetic Declination

Magnetometer

Measure Earth's magnetic field

- Depends on location, time
- Distortion due to metal objects or nearby EM fields

Examples



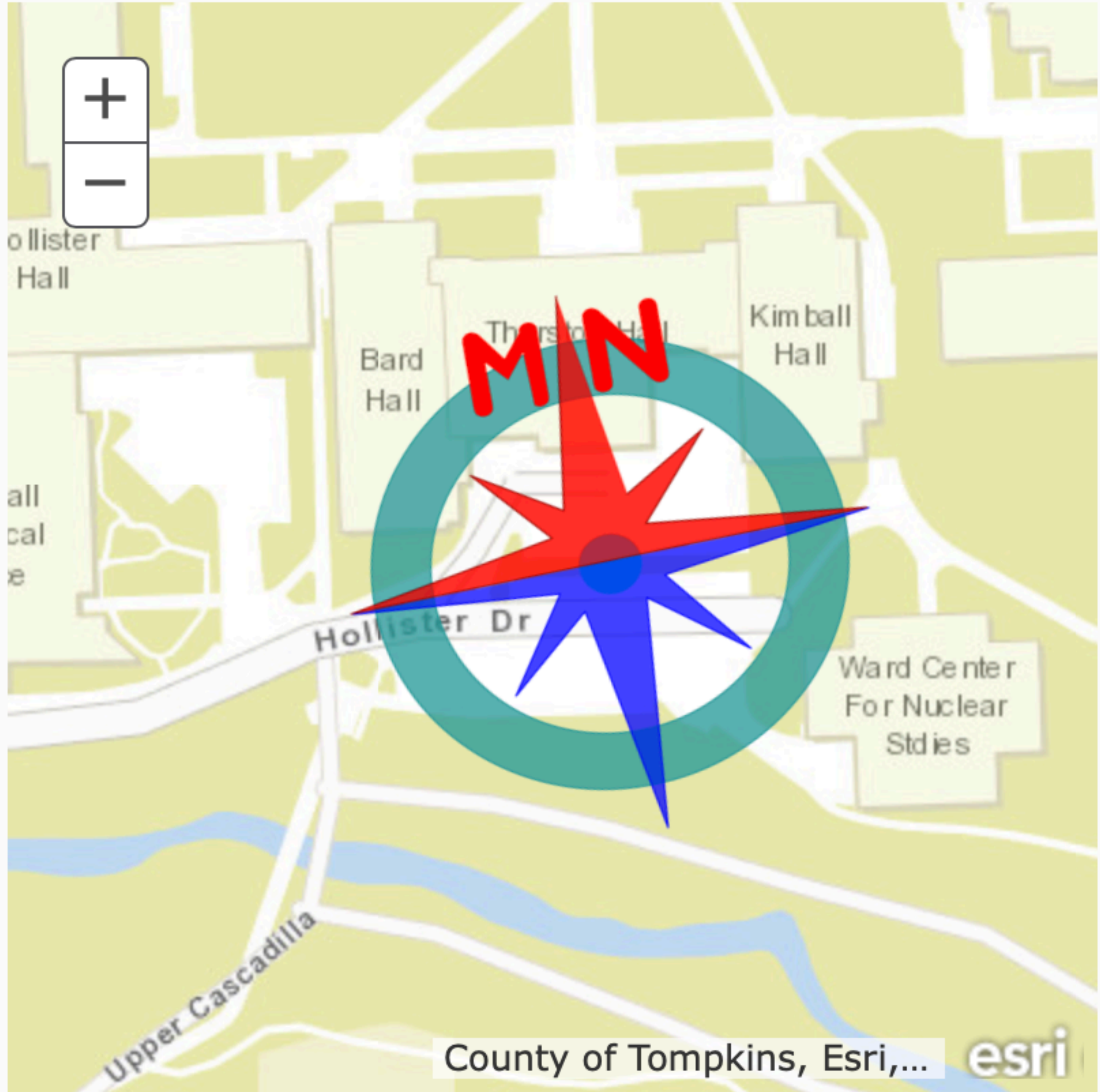
Magnetometer

Measure Earth's magnetic field

- Magnetic North is along the xmax-axis

Model Used:	WMMHR-2025						
Latitude:	42° 26' 37" N						
Longitude:	76° 29' 1" W						
Elevation:	0.0 km Mean Sea Level						
Date	**Declination (+ E | - W)**	**Inclination (+ D | - U)**	**Horizontal Intensity**	**North Comp (+ N | - S)**	**East Comp (+ E | - W)**	**Vertical Comp (+ D | - U)**	**Total Field**
2026-01-27	-11° 35' 31"	67° 31' 32"	19,930.1 nT	19,523.6 nT	-4,004.8 nT	48,176.1 nT	52,135.8 nT
Change/year	0° 2' 26"/yr	-0° 5' 57"/yr	41.6 nT/yr	43.6 nT/yr	5.5 nT/yr	-135.4 nT/yr	-109.2 nT/yr
Uncertainty	0° 22'	0° 11'	130 nT	135 nT	85 nT	134 nT	134 nT

Model Used:	WMMHR-2025
Latitude:	42° 26' 37" N
Longitude:	76° 29' 1" W
Date	**Declination**
2026-01-27	11° 36' W ± 0° 22' changing by 0° 2' E per year

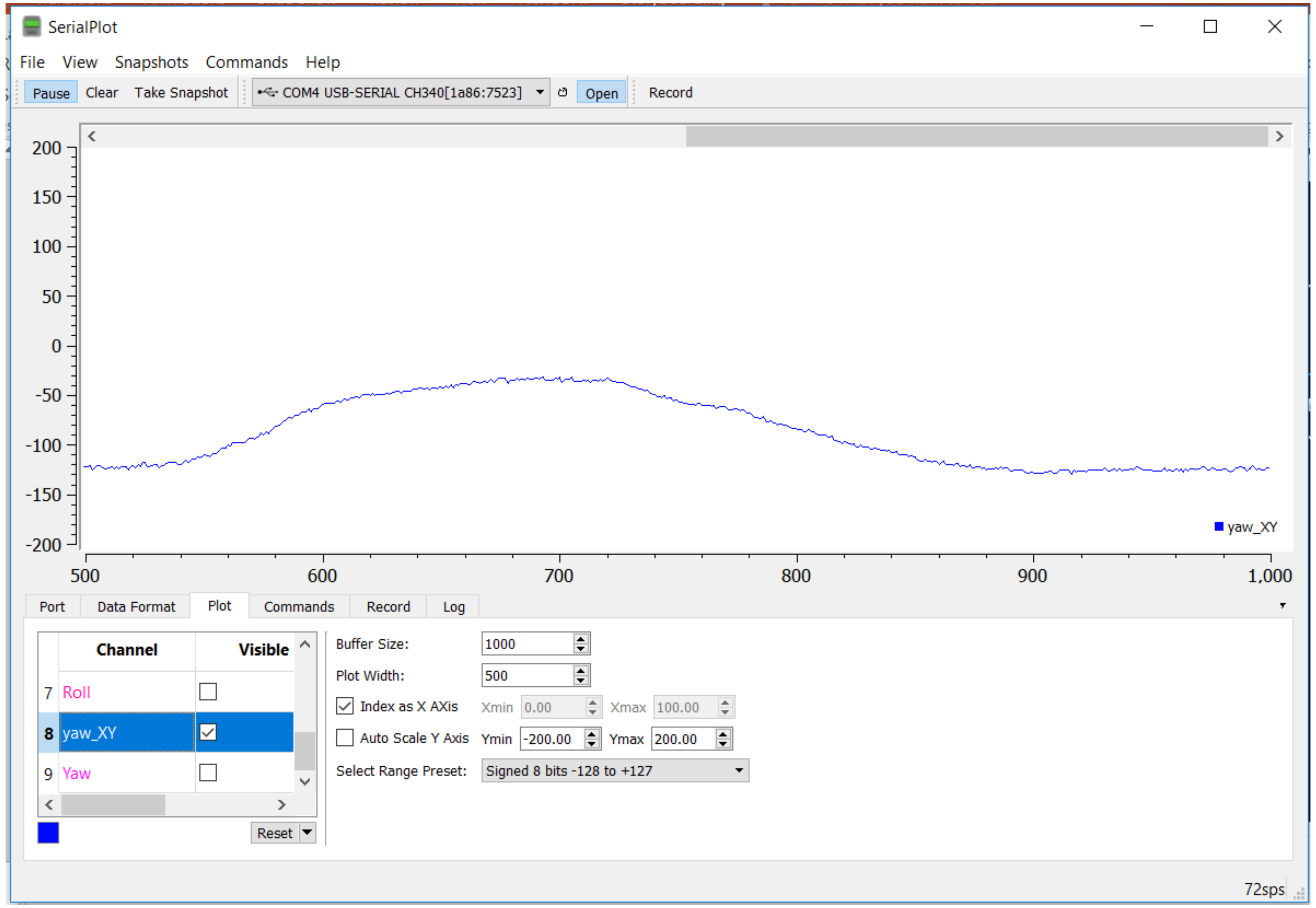


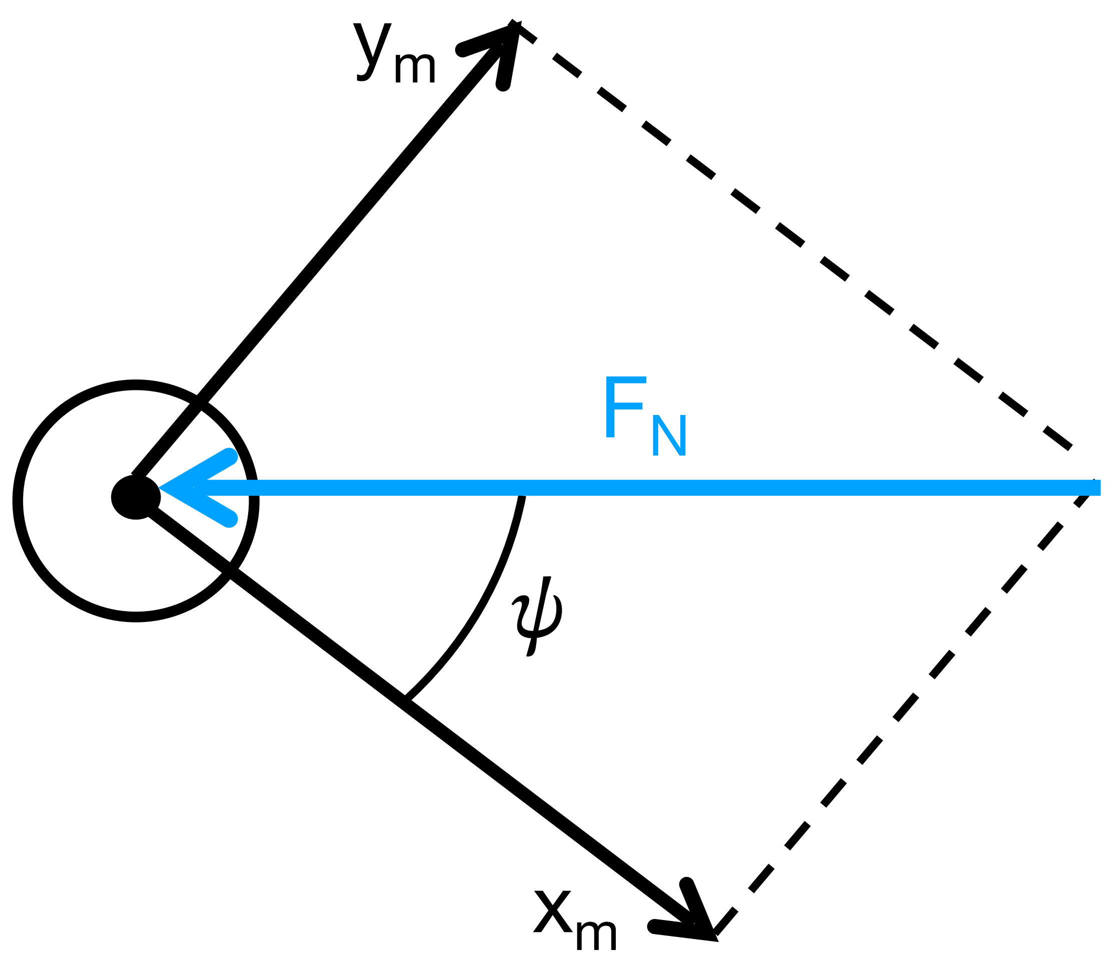
https://www.ngdc.noaa.gov/geomag/calculators/magcalc.shtml#declination

Magnetometer

Measure Earth's magnetic field

- $\psi = \text{atan2}(x_m, y_m)$





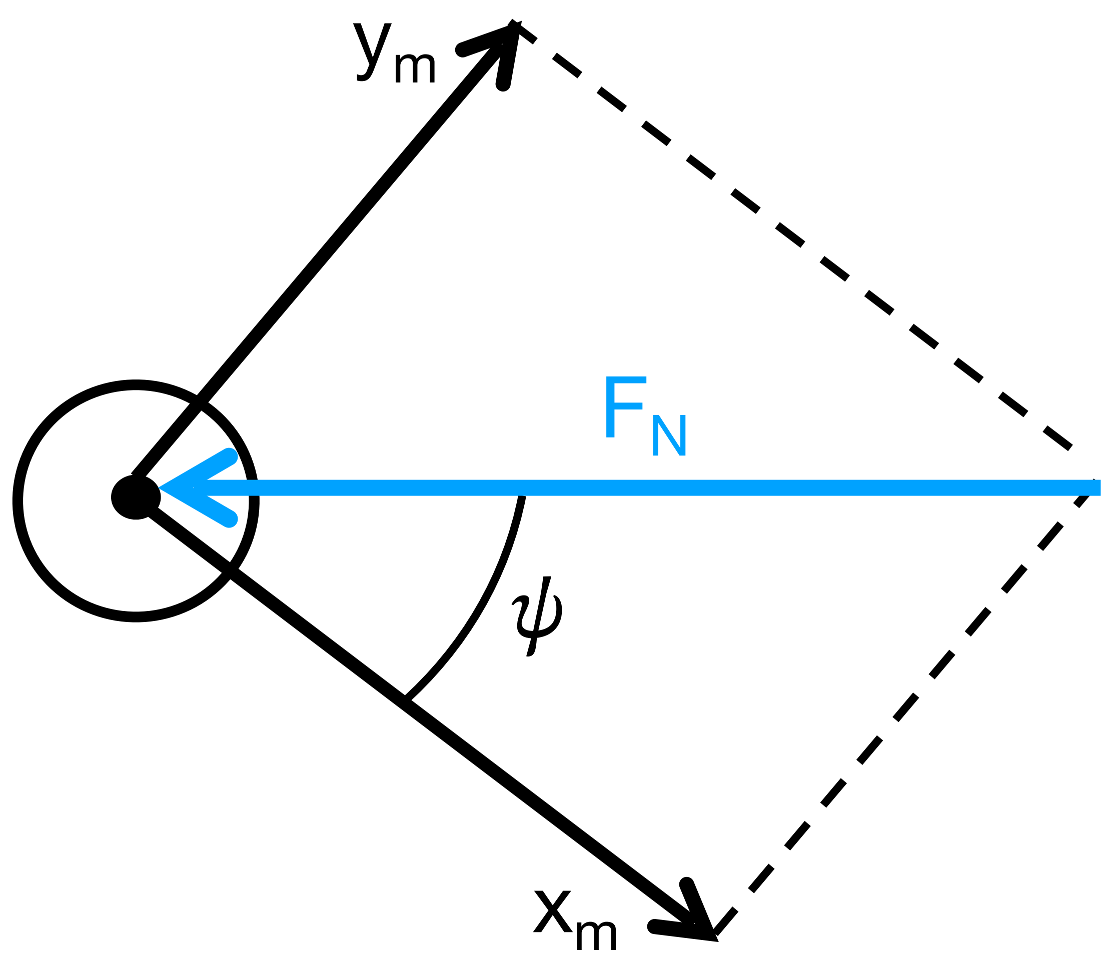
Magnetometer

Measure Earth's magnetic field

- $\psi = \mathrm{atan2}(x_m, y_m)$
- How to compensate for tilt? — Fuse data

$$\begin{bmatrix} x_m \\ y_m \\ z_m \end{bmatrix} = R_{x,\phi} R_{y,\theta} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = R^T_{x,\phi} R^T_{y,\theta} \begin{bmatrix} x_m \\ y_m \\ z_m \end{bmatrix} = \begin{bmatrix} c_\theta & 0 & c_\phi - s_\theta \\ s_\phi s_\theta & c_\phi & c_\theta s_\phi \\ c_\phi s_\theta & -s_\phi & c_\phi c_\theta \end{bmatrix} \begin{bmatrix} x_m \\ y_m \\ z_m \end{bmatrix}$$



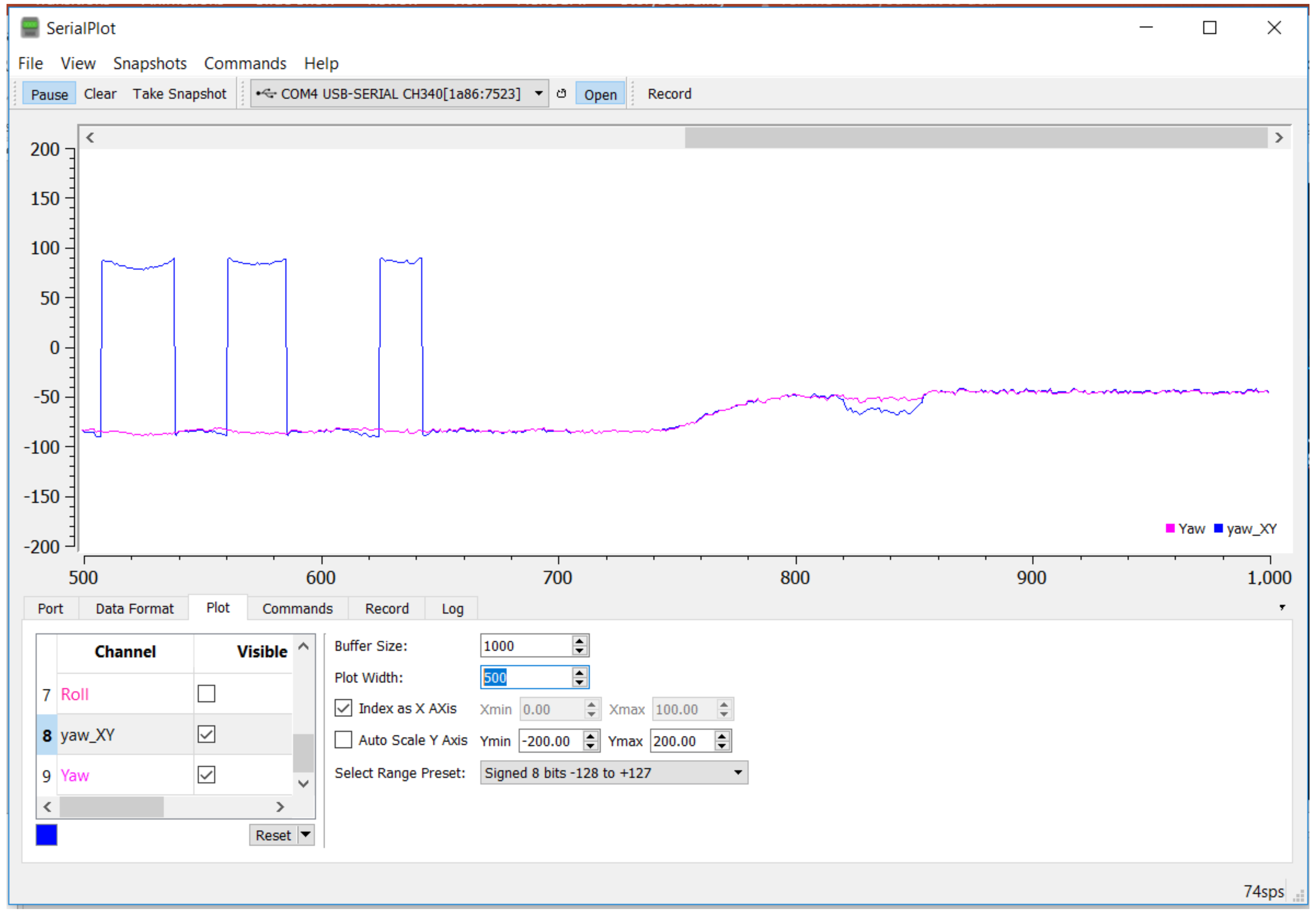
$$x = y_m \cos(\phi) - z_m \sin(\phi)$$
$$y = x_m \cos(\theta) + y_m \sin(\phi)\sin(\theta)$$
$$+ z_m \cos(\phi)\sin(\theta)$$

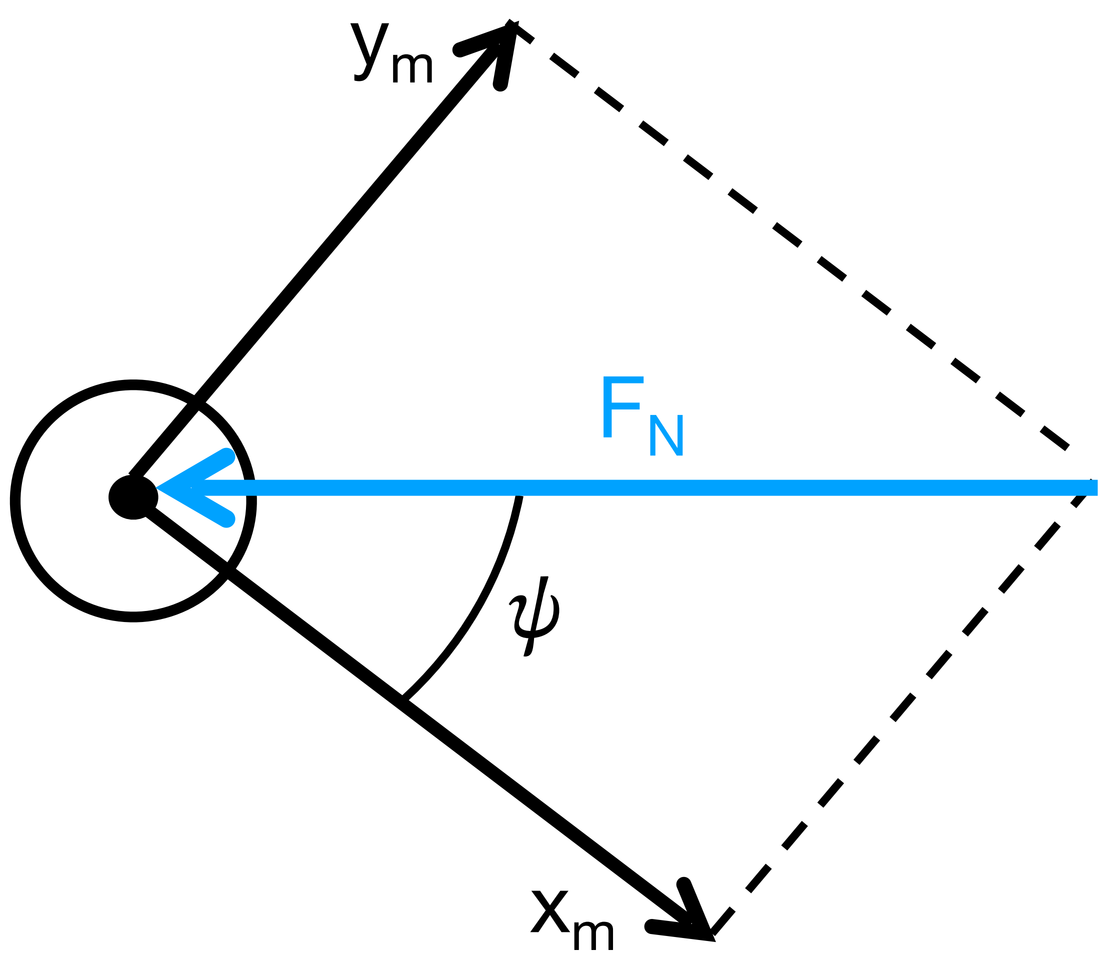
$$\psi = \mathrm{atan2}(x, y)$$

Magnetometer

Measure Earth's magnetic field

- $\psi = \mathrm{atan2}(x_m, y_m)$
- How to compensate for tilt? — Fuse data





Sources and references

- http://www.chrobotics.com/library/accel-position-velocity
- EE 267 Virtual Reality, by Gordon Wetzstein at Stanford University
- Analog.com
- https://toptechboy.com/
- Prof. Kirstin Petersen

Class Action Items

- If you want to drop the class, please let me know **ASAP** and return your lab kits!
- **January 30th, midnight**: Make a GitHub repository and build your Github page
    - Include: name, photo, a small introduction, and the class number
    - Share **the page link** in the canvas assignment
- **February 3rd (8am)** for Lab 401, and **February 4th (8am)** for Labs 402 & 403: Lab 1A and Lab 1B write-ups are due!