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ECE 4160/5160
MAE 4910/5910

Fast Robots

Lab 6

Probability and Bayes Theorem

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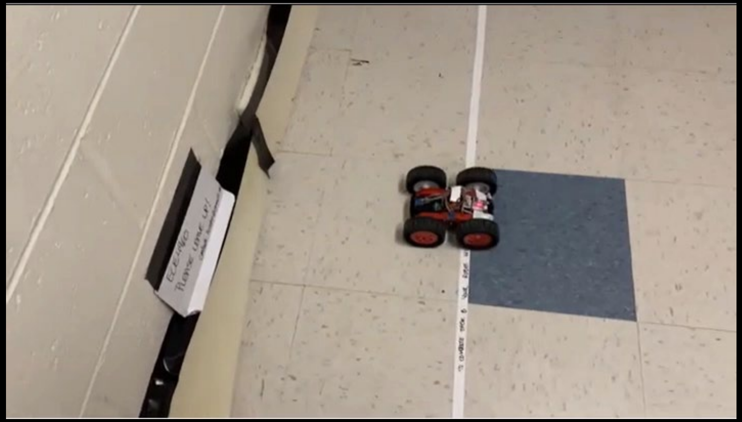
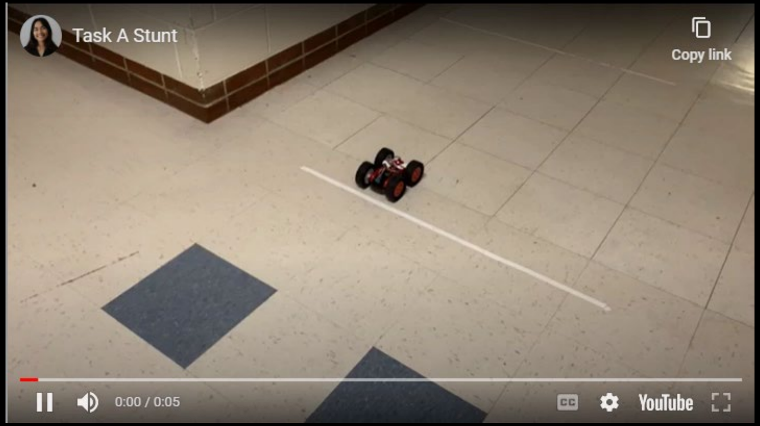
Lab Prep

Lab 6: PID control



Lab 7: Sensor Fusion

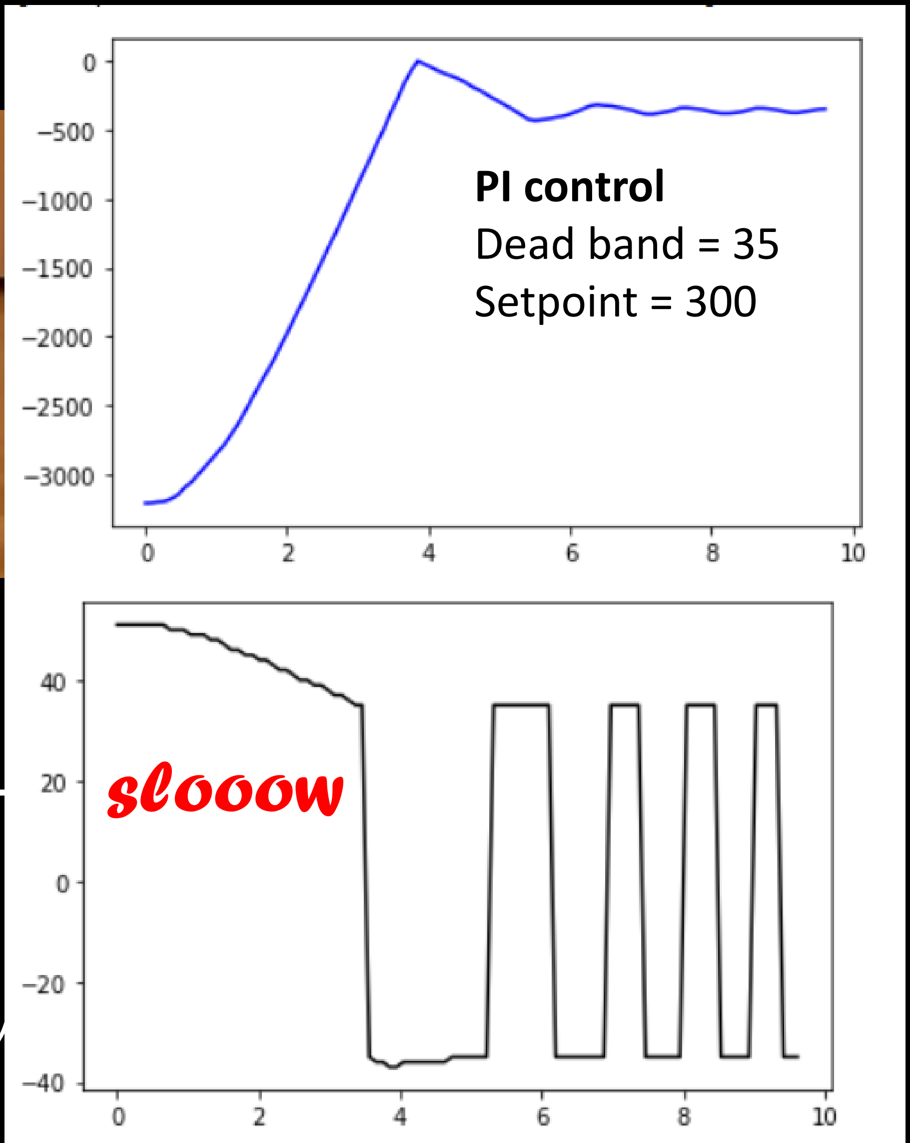
Lab 8: Stunt

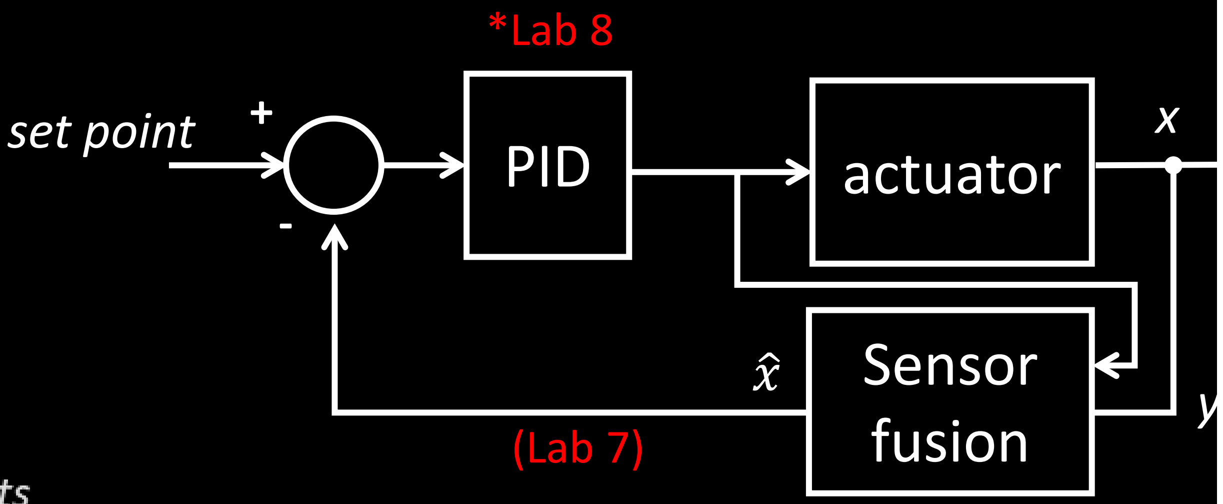


Credit: Anya Prabowo, 2022

Lab 6: PID control

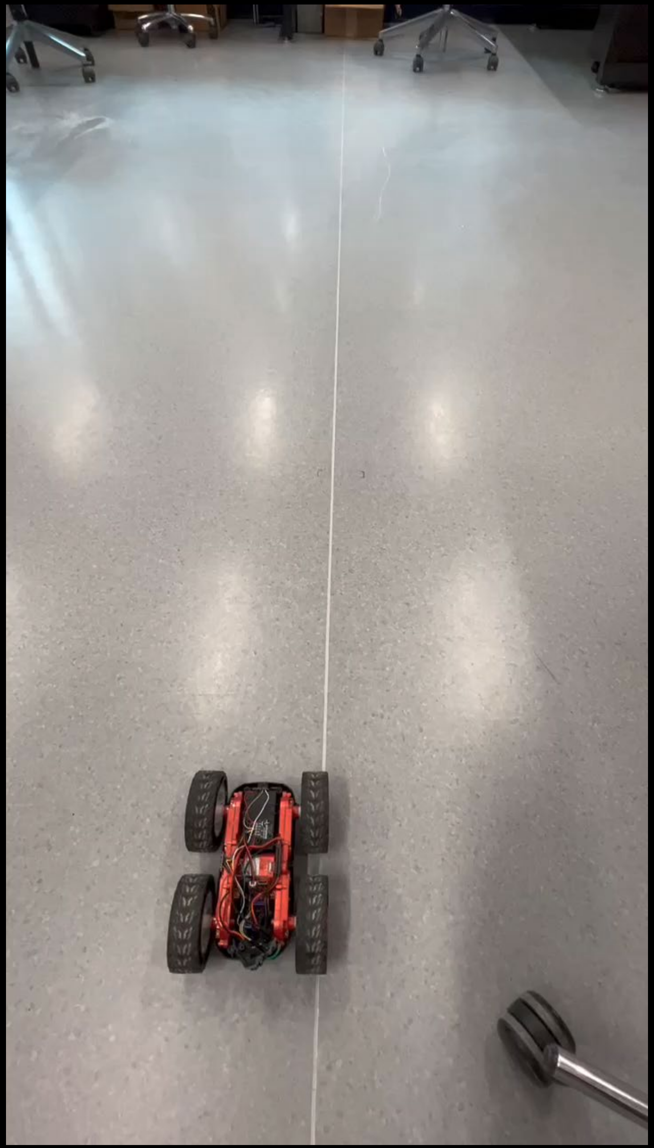
- Task A: Position control
  - Benefit: Easiest

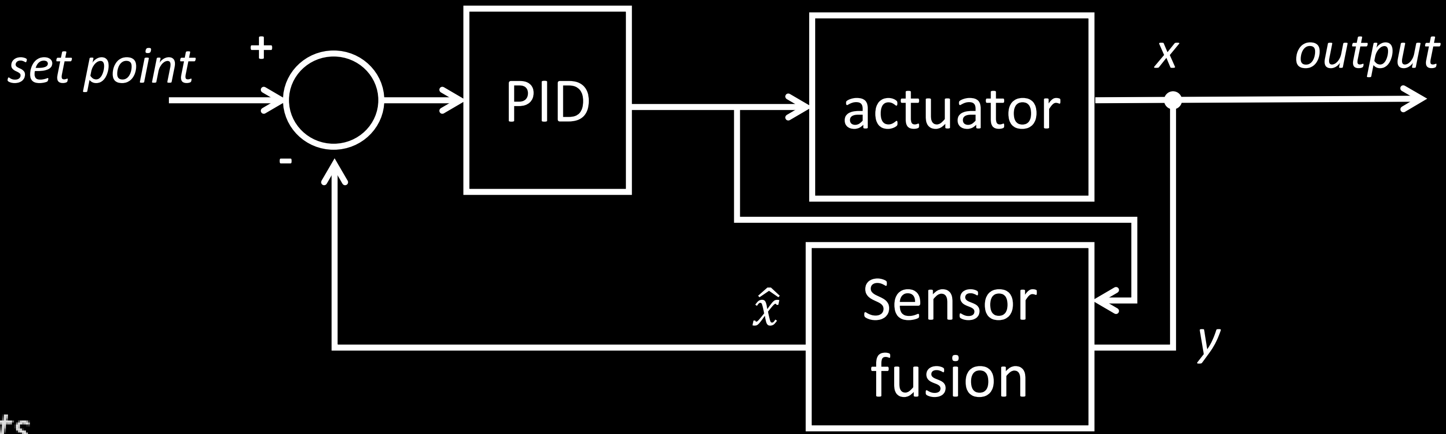




Lab 6: PID control

- Task A: Position control
- Task B: Orientation control
  - Benefit: Good start to lab 9





cei-lab.github.io/FastRobots-2023/Lab6.html

FastRobots-2023

ECE4160/5160-MAE 4190/5190: Fast Robots course, offered at Cornell University in Spring 2023

View On GitHub

This project is maintained by CEI-lab

Hosted on GitHub Pages using the Dinky theme

Fast Robots @Cornell, Spring 2023

Return to main page

Lab 6: Closed-loop control (PID)

Objective

The purpose of this lab is to get experience with PID control. The lab is fairly open ended, you can pick whatever controller works best for your system. 4000-level students can choose between P, PI, PID, PD; 5000-level students can choose between PI and PID controllers. Your hand-in will be judged upon your demonstrated understandi... your solution.

This lab is part of a series ... choose to do either positio... very strained for time. The... fun!). Whatever you choos... for improving/speeding up... the coming weeks.

Good examples from last year:

- Orientation control
  - https://kr397.github.io/ece4960-labs/lab6.html
- Position control
  - https://bwagner2-git.github.io/lab6

Lab 6-8: PID control – Sensor Fusion - Stunt

- Task A: Position control
- Task B: Orientation control

Procedure

- Lab 6: Get basic PID to work
- Do the pre-lab: you need good debugging scripts
- Start simple and work your way up, then hack away…
  - Start slow (sampling rates, control frequency)
  - Avoid blocking statements
- Wind-up, derivative LPF, derivative kick
- Motor scaling function
  - Range of analogWrite: [0;255]
  - Directionality
  - Deadband

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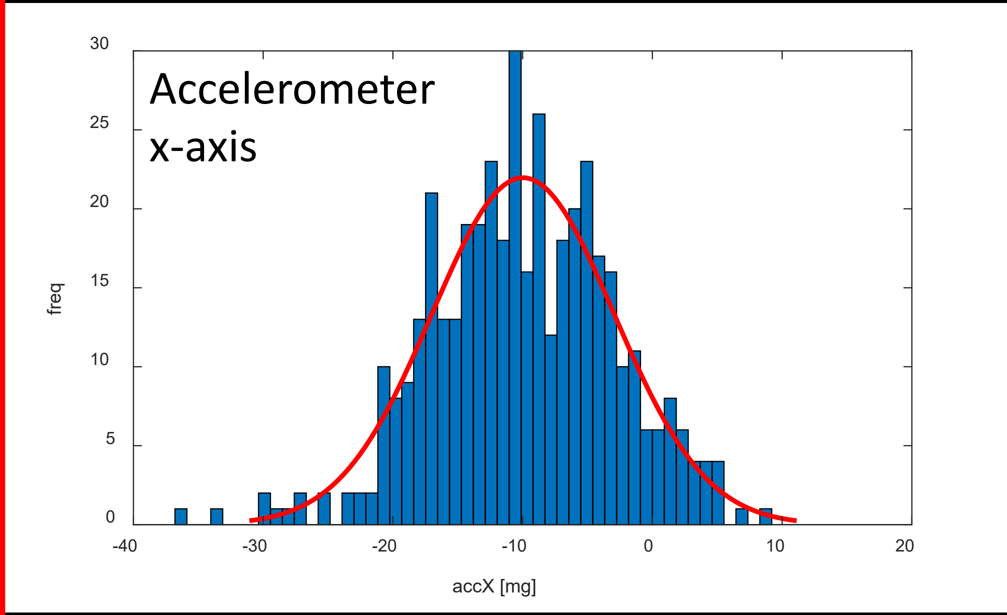
Fast Robots

Probability and Bayes Theorem

Recap from ECE 3100 Intro to Probability and Inference

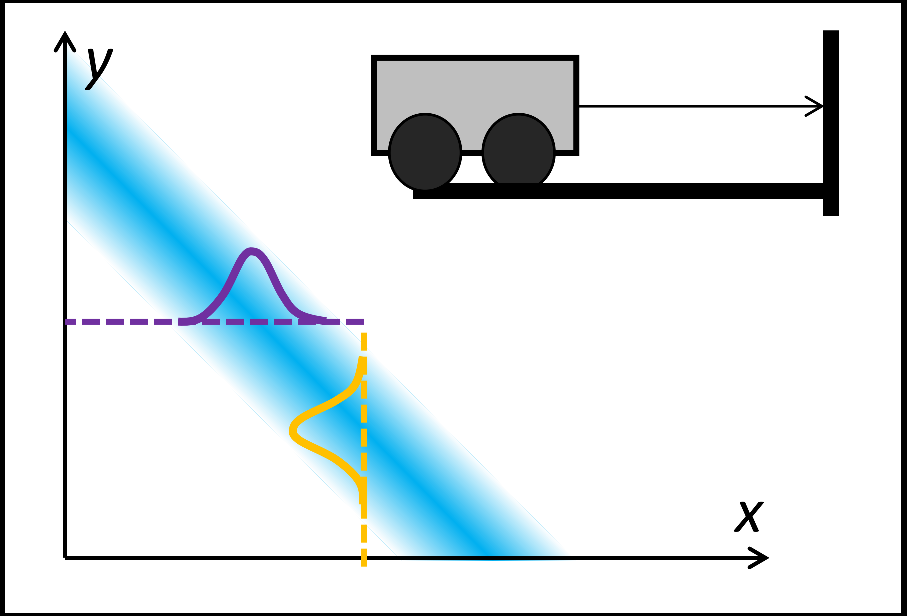
- Random variable
  - $X\colon \Omega \rightarrow \mathbb{R}$
- The probability that the random variable X has value x
  - $P(X = x)\ or\ p(x)$
- Probabilities sum to 1
  - $\sum_x P(X = x) = 1$
- Probabilities are always greater than 0
  - $P(X{=}x) \geq 0$
- Joint distribution Y
  - $p(x, y) = P(X = x\ and\ Y = y)$
- Conditional probability
  - $p(x|y) = \frac{p(x,y)}{p(y)}$

- Mean
  - μ = -9.97306mg
- std dev
  - σ =7.0318mg
- Variance
  - σ^2
- Gaussian distributions
  - $[\mu \mp \sigma]$
  - Symmetric
  - Unimodal
  - Sum to "unity"



Conditional probability

- $p(x|y) = \frac{p(x,y)}{p(y)}$
- Robot/sensor example
- Exercise
  - Two children, the older is female, what is the probability that the second child is female?
    - 50%
  - Two children, one is female, what is the probability that the second child is female?
    - 33%
    - F-M, F-F, M-F, (M-M)



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- Conditional probability
  - $p(x|y) = \frac{p(x,y)}{p(y)}$
- Marginal probability
  - $p(x) = \sum_y p(x|y)p(y)$

- Independence
  - $p(x, y) = p(x)p(y)$
  - $p(x|y) = p(x) = \dfrac{p(x, y)}{p(y)}$ (Coin example)
- If X and Y are conditionally independent given Z=z, then
  - $p(x, y|z) = p(x|z)p(y|z)$

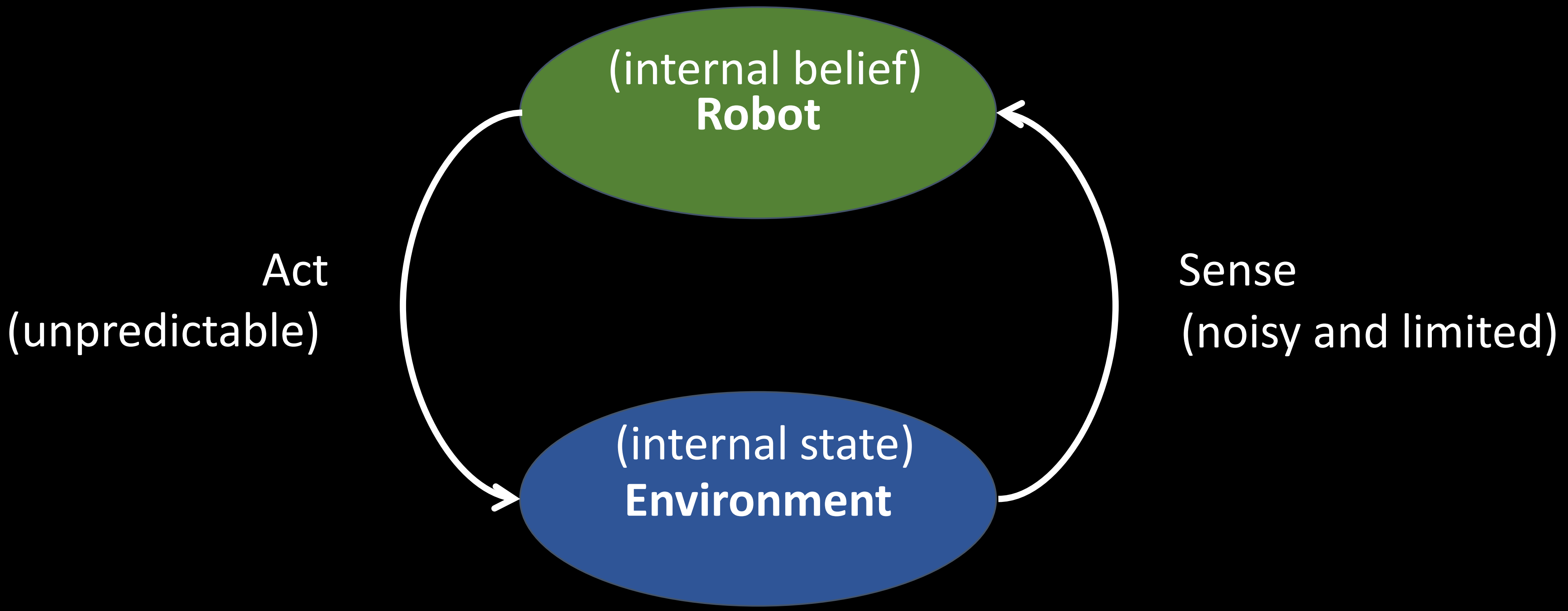
Why consider uncertainty?

- Uncertainty is inherent in the world
- Five major factors
  - Unpredictable environments
  - Sensors
    - Subject to physical laws
    - Signal to noise ratio
  - Robot motion
    - Noise, wear and tear, battery state, etc.
    - Accuracy versus cost
  - Models
    - Abstractions of the real world
  - Computation
    - Real time systems
    - Timely response versus accuracy

Exercise

- Is this dress black and royal blue, or white and gold?
- Where does the uncertainty come from?
  - blue and black under a yellow-tinted illumination (left)
  - white and gold under a blue-tinted illumination (right)

Robot-Environment Model

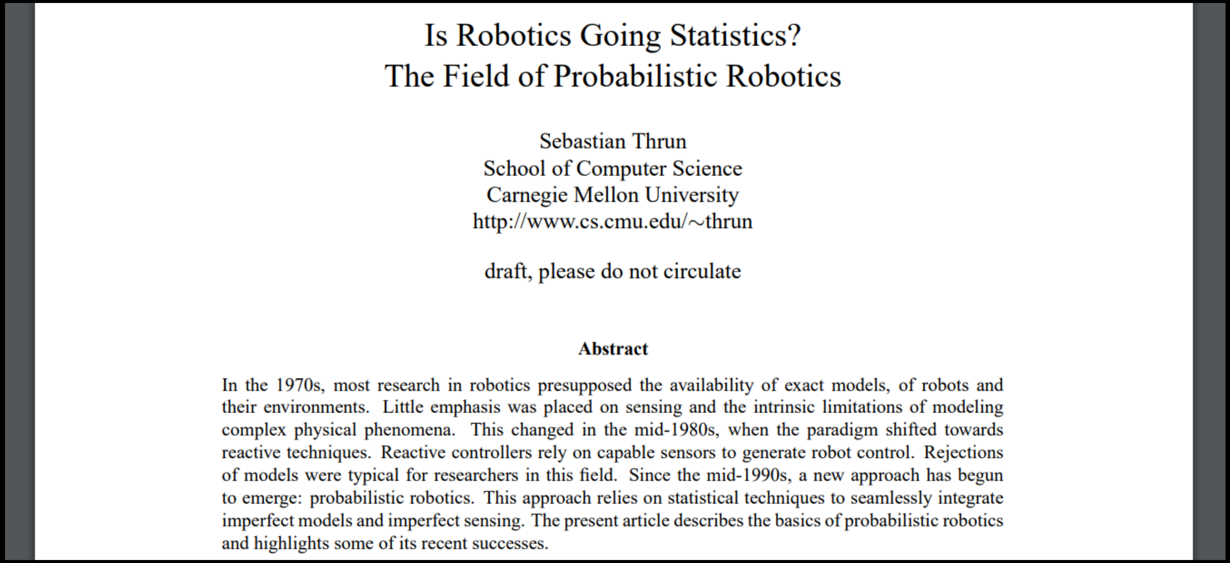


Probabilistic Approach

"A robot that carries a notion of its own uncertainty and that acts accordingly is superior to one that does not."

- Probabilistic Robotics by Thrun, Burgard, Fox

- Probabilistic approaches in contrast to traditional model-based motion planning techniques or reactive behavior-based motion:
  - tend to be more robust to sensor and model limitations
  - weaker requirements on the accuracy of the robot's models



Is Robotics Going Statistics?
The Field of Probabilistic Robotics

Sebastian Thrun
School of Computer Science
Carnegie Mellon University
http://www.cs.cmu.edu/~thrun

draft, please do not circulate

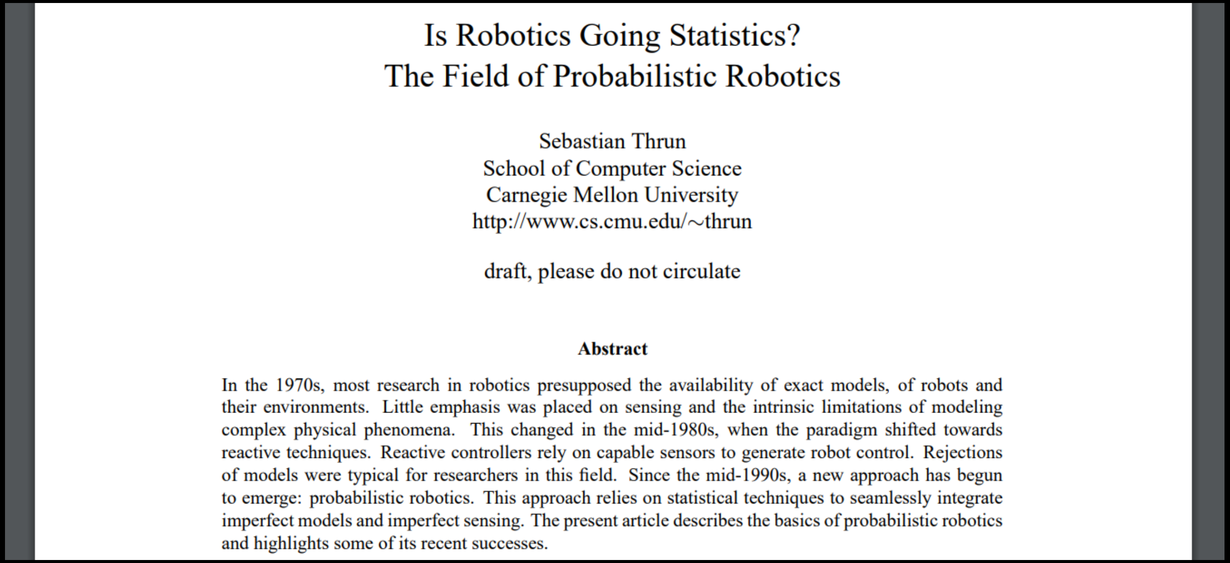
Abstract

In the 1970s, most research in robotics presupposed the availability of exact models, of robots and their environments. Little emphasis was placed on sensing and the intrinsic limitations of modeling complex physical phenomena. This changed in the mid-1980s, when the paradigm shifted towards reactive techniques. Reactive controllers rely on capable sensors to generate robot control. Rejections of models were typical for researchers in this field. Since the mid-1990s, a new approach has begun to emerge: probabilistic robotics. This approach relies on statistical techniques to seamlessly integrate imperfect models and imperfect sensing. The present article describes the basics of probabilistic robotics and highlights some of its recent successes.

Probabilistic Approach

+ Explicitly represent the uncertainty using probability theory
+ Accommodate inaccurate models
+ Accommodate imperfect sensors
+ Robust in real-world applications
+ Best known approach to many hard robotics problems

- Computationally demanding
- Need to approximate
- False assumptions



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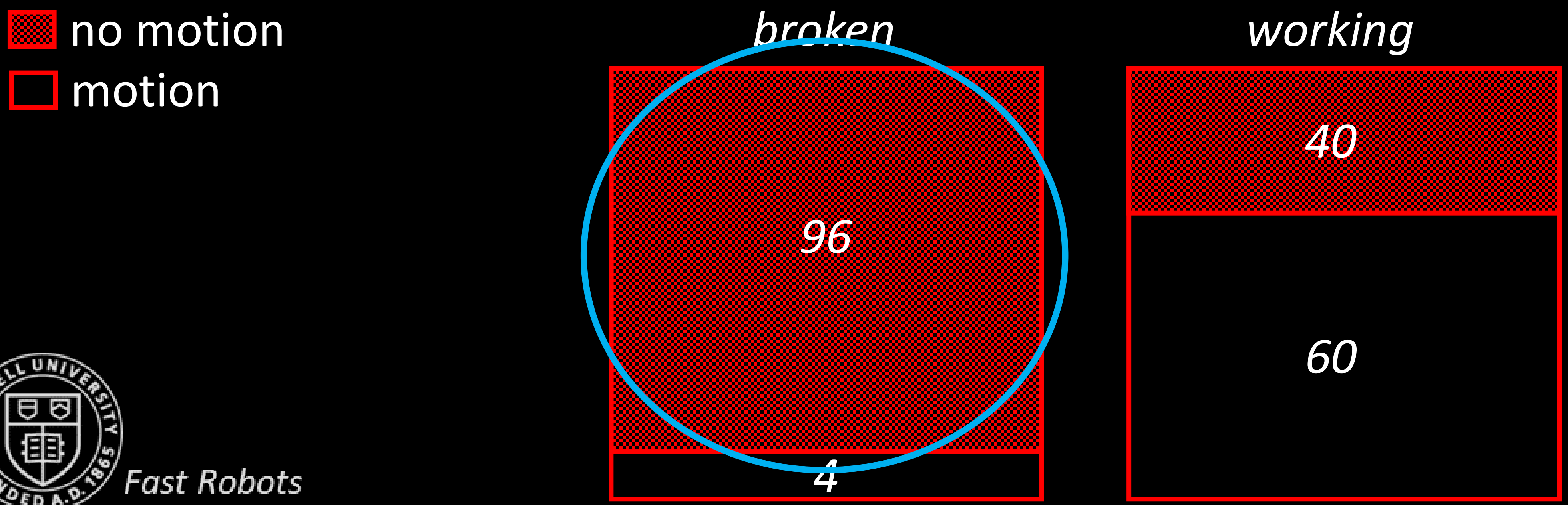
Bayesian Inference

- Inference = educated guessing
- Bayesian inference = guessing in the style of Bayes
- Example
  - *EdDiscussion:* "My robot stopped moving, the hardware is broken, send me new parts"
  - What is the probability that the robot is broken, given that it stopped moving?



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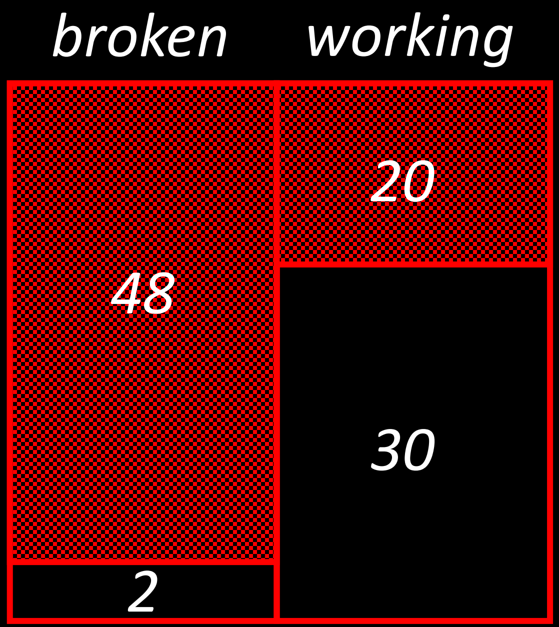
no motion
motion

After lab 8



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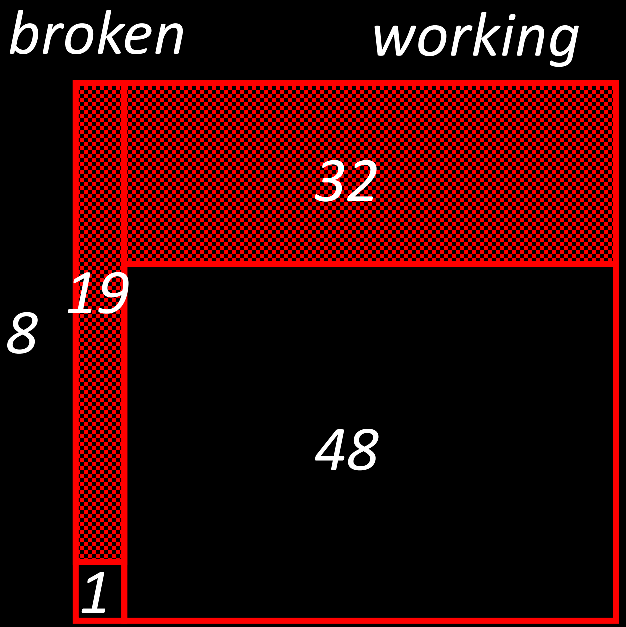

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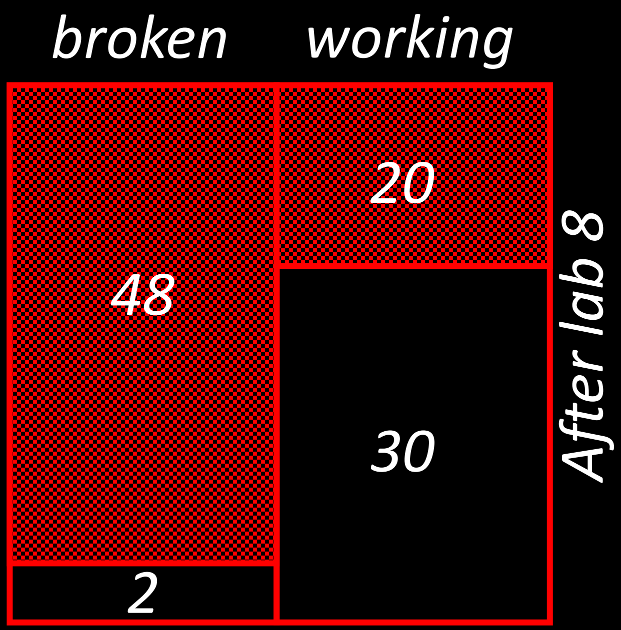
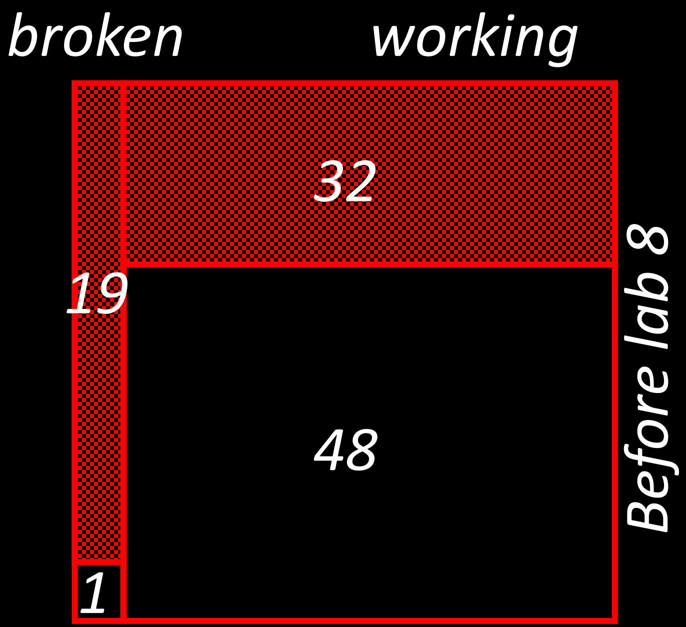
- **Translate to probability**
  - P(something) = #something / #everything
  - Before lab 8:
    - P(broken) = #broken / #kits = 20 / 100 = 0.2
    - P(working) = #working / #kits = 80 / 100 = 0.8
  - After lab 8:
    - P(broken) = #broken / #kits = 50 / 100 = 0.5
    - P(working) = #working / #kits = 50 / 100 = 0.5





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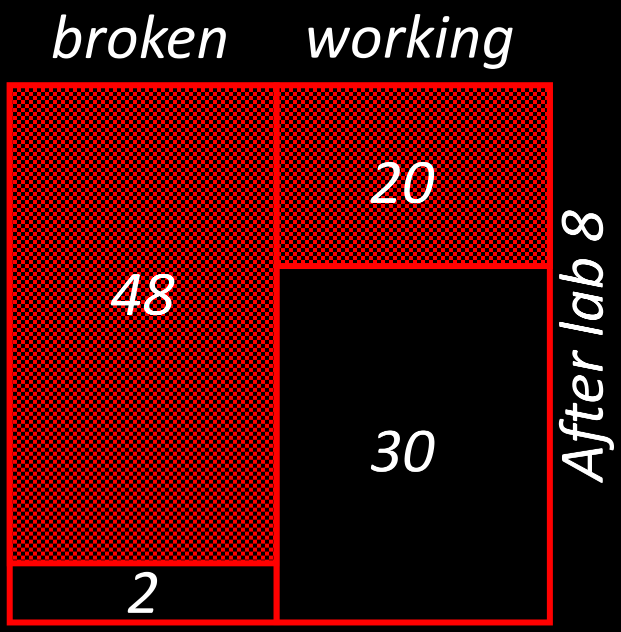
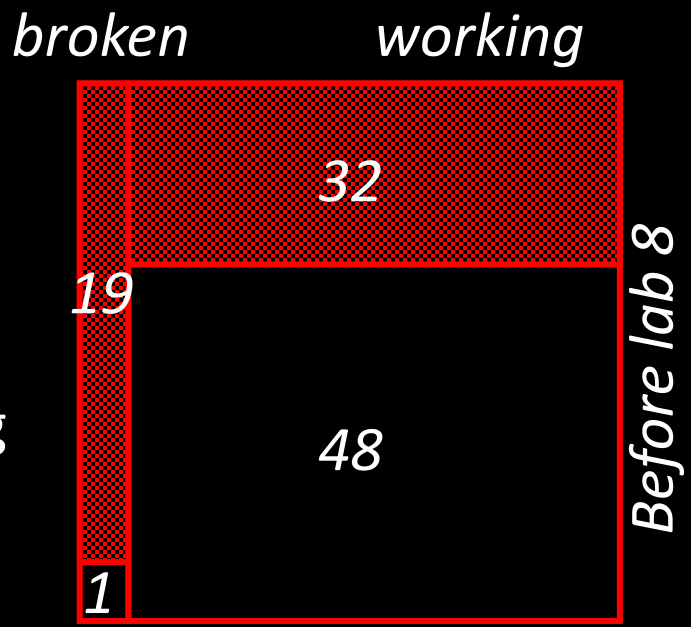


- **Conditional Probability**
  - If you know that the robot is broken, what is the probability that it stopped moving?
  - P(no motion | broken) = #broken and no motion / #broken
  - After lab 8 = 48/50 = 0.96
  - P(no motion | working) = #working and no motion / #working
  - After lab 8 = 20/50 = 0.40



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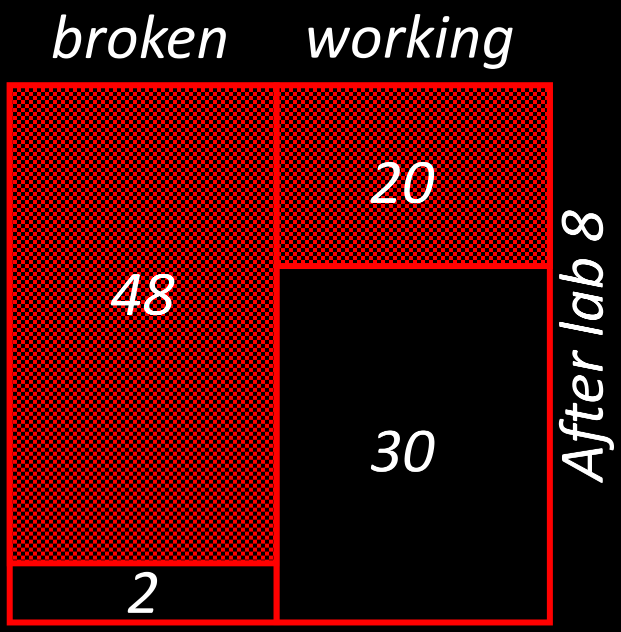
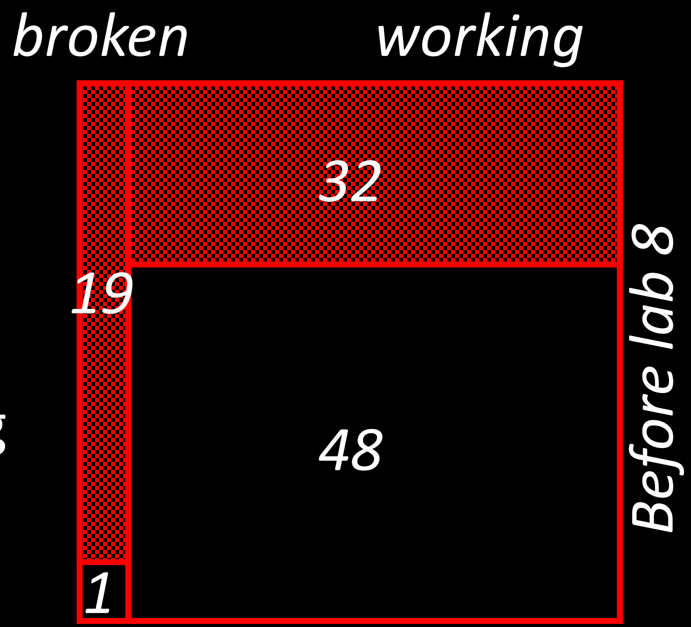


- **Conditional Probability**
  - If you know that the robot is broken, what is the probability that it stopped moving?
  - P(no motion | broken) = #broken and no motion / #broken
  - Before lab 8 = 19/20 = 0.96
  - P(no motion | working) = #working and no motion / #working
  - Before lab 8 = 32/80 = 0.40



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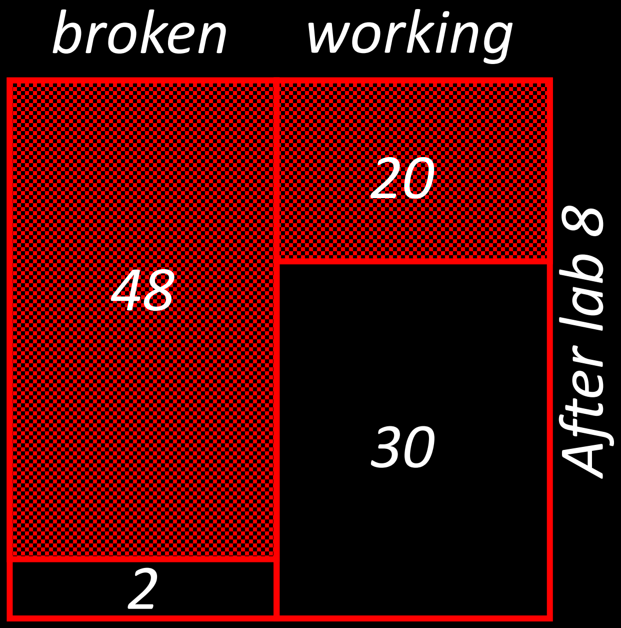
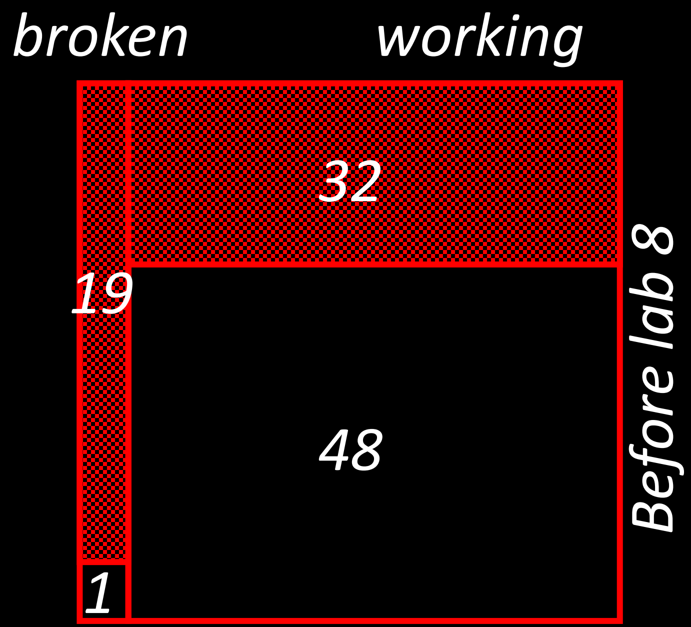
- **Conditional Probability**
  - If you know that the robot is broken, what is the probability that it stopped moving?
  - P(A|B) is the probability of A, given B
  - Note: P(A|B) is not equal to P(B|A)
    - P(cute|puppy) ≠ P(puppy|cute)





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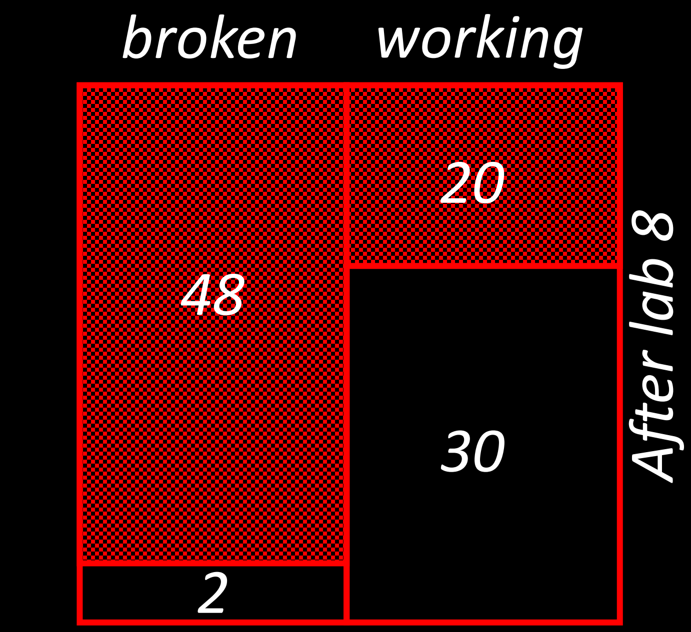
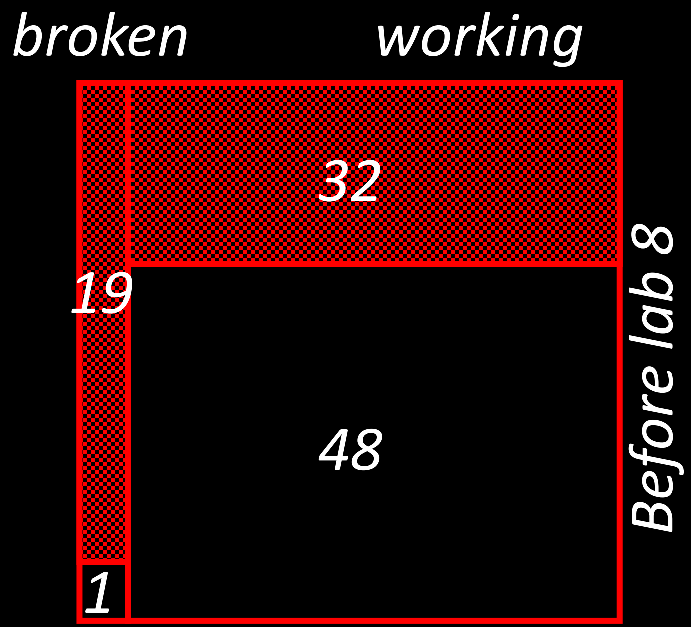


- **Joint Probability**
  - What is the probability that the robot is both broken and not moving?
  - After lab 8:
    - P(broken and not moving)
      = P(broken)*P(not moving | broken)
      = 0.5 * 0.96 = 0.48



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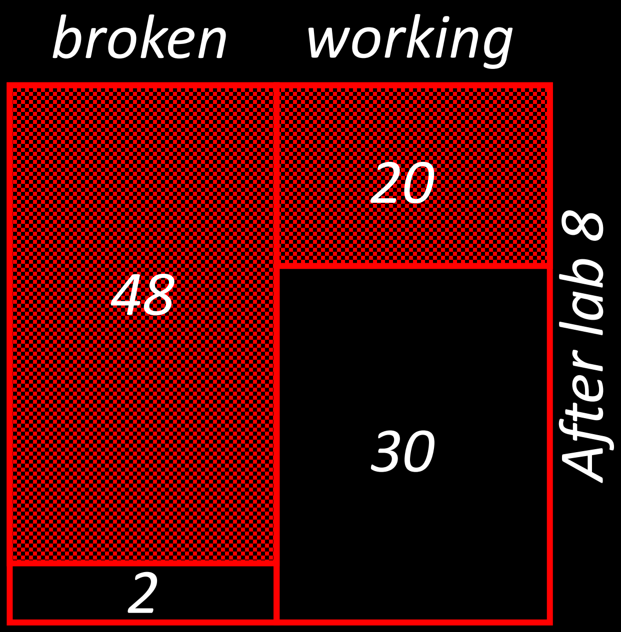
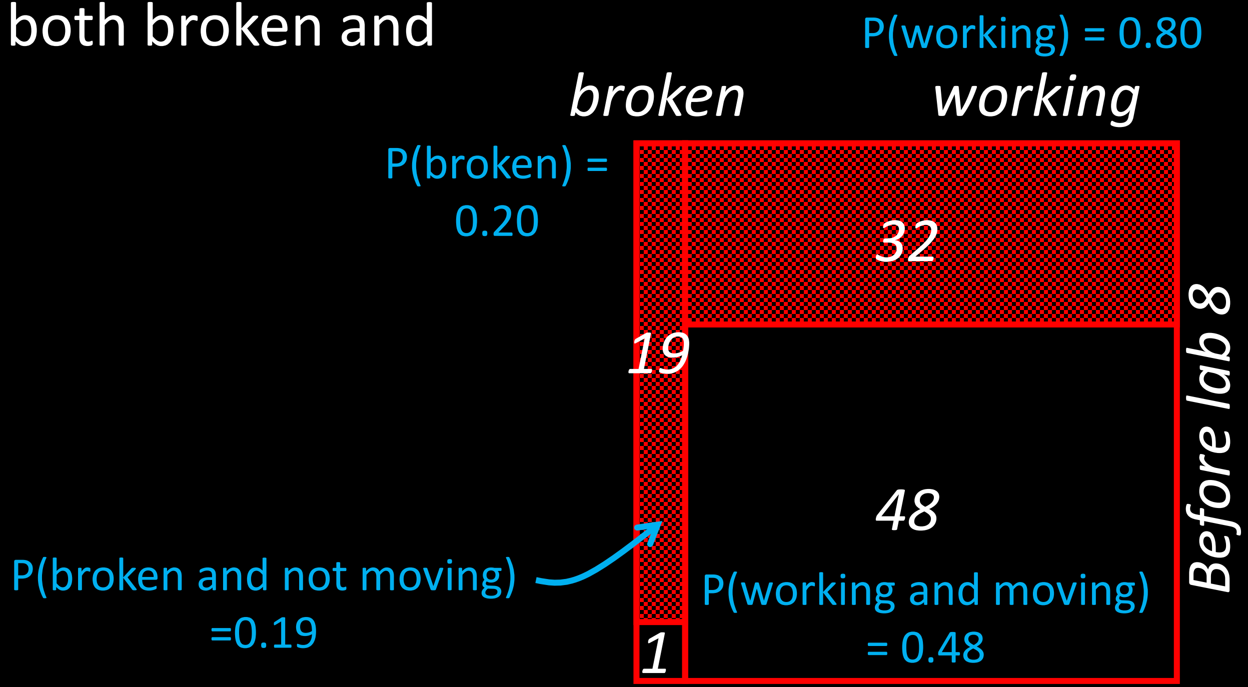
- **Joint Probability**
  - What is the probability that the robot is both broken and not moving?
  - P(broken and not moving)
    = P(broken)*P(not moving | broken)
    = 0.20 * 0.96 = 0.192
  - P(working and moving)
    = P(working)*P(moving | working)
    = 0.80 * 0.60 = 0.48





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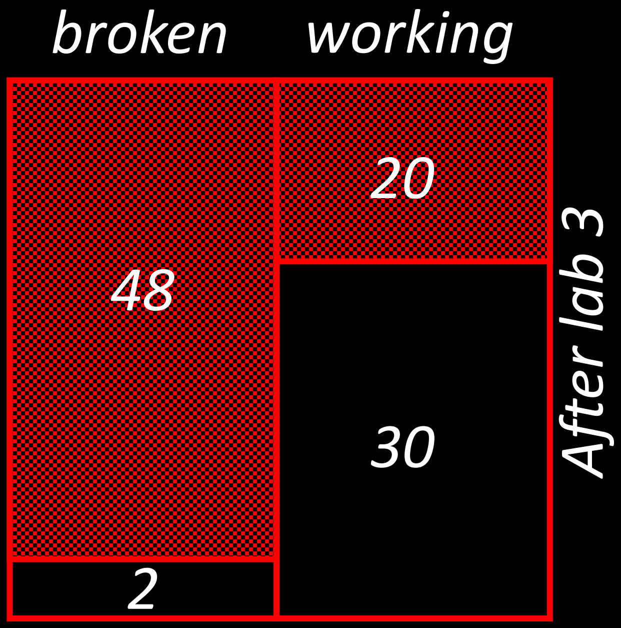
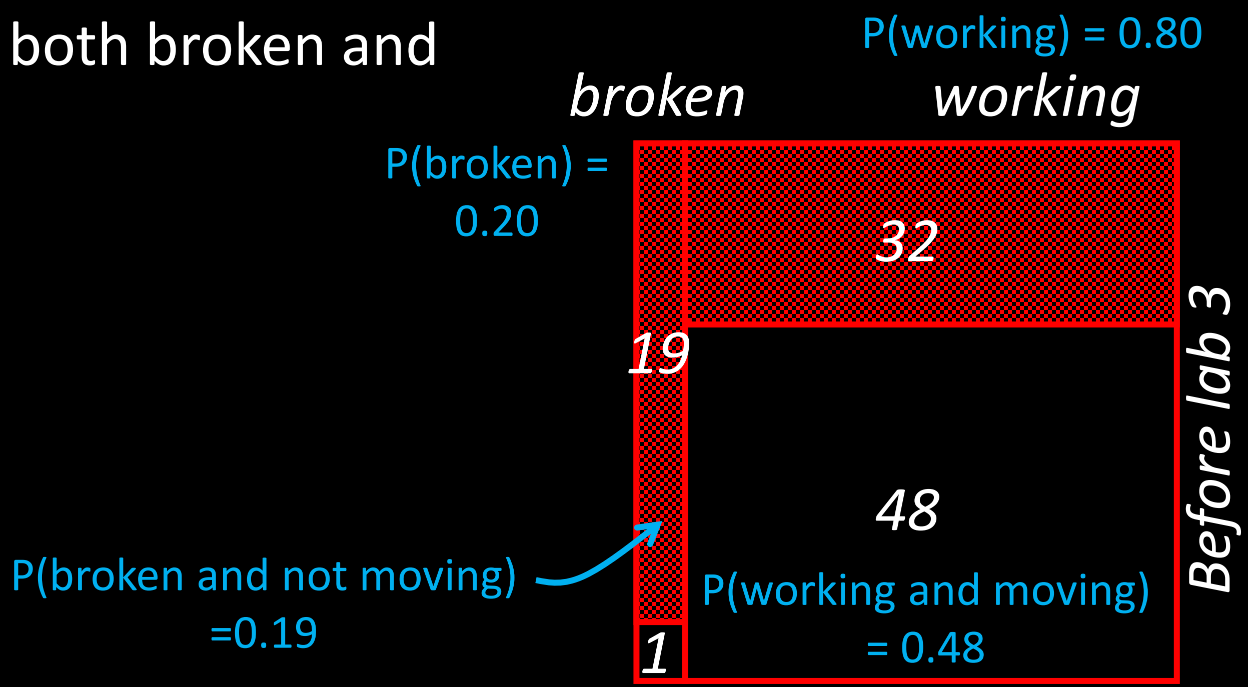
- **Joint Probability**
  - What is the probability that the robot is both broken and not moving?
  - P(A, B) = P(A∩B) = P(A and B)
  - P(A, B) = P(A)*P(B|A)
  - P(A, B) = P(B, A)





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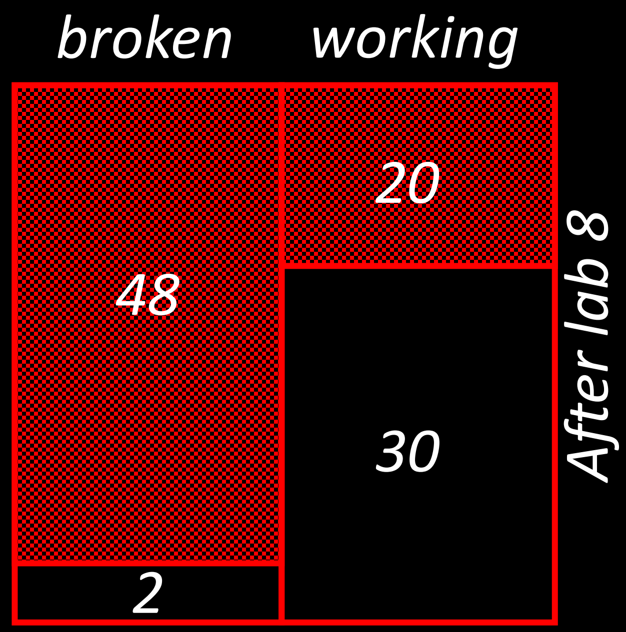
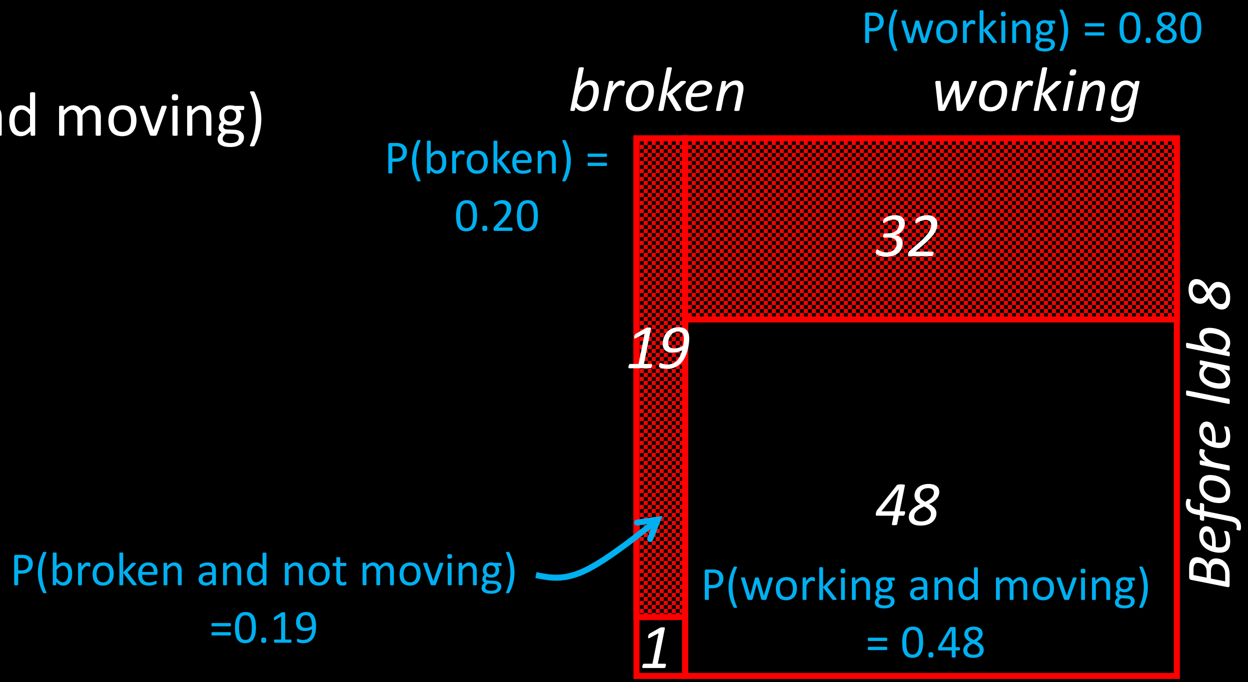
- **Marginal Probability**
  - P(moving)
    = P(broken and moving) + P(working and moving)
    = 1/100 + 48/100 = 0.49
  - P(not moving)
    = 19/100 + 32/100 = 0.51





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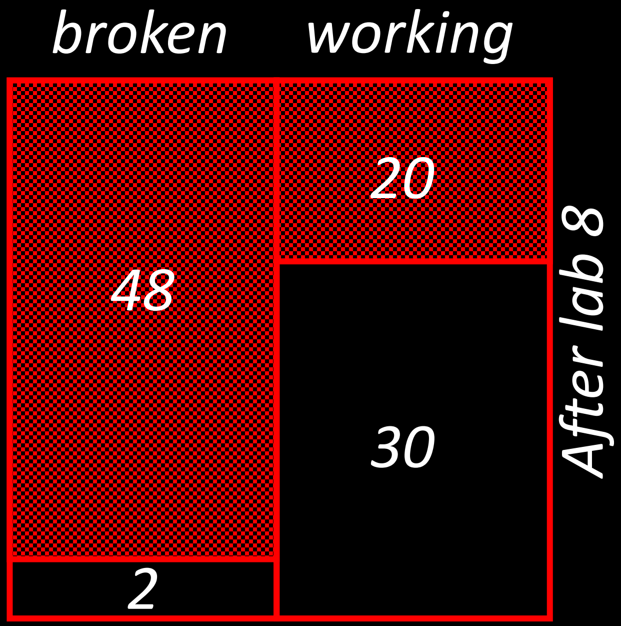
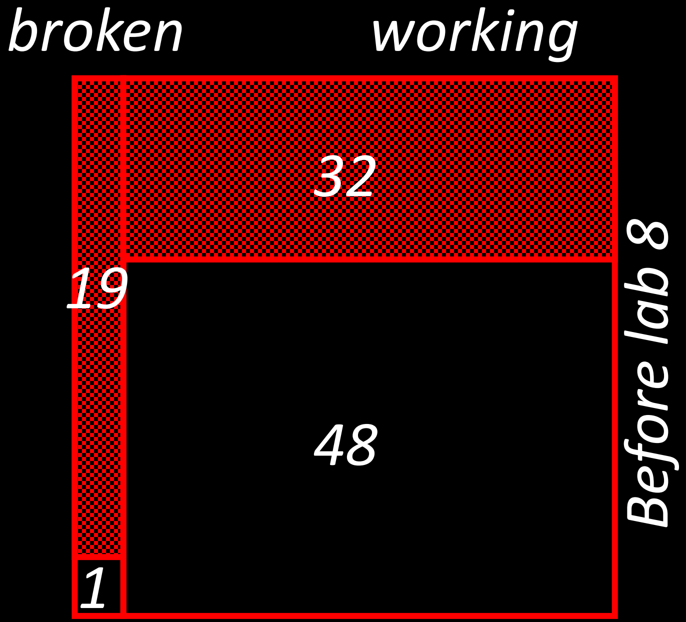
- Example
  - *EdDiscussion:* "My robot stopped moving, the hardware is broken, send me new parts"
  - What is the probability that the robot is broken, given that it stopped moving?
  - P(broken | not moving) = ???
- P(broken and not moving)
  = P(not moving)*P(broken|not moving)
- P(not moving and broken)
  = P(broken)*P(not moving|broken)
- P(broken|not moving) = $\frac{\text{P(broken)*P(not moving|broken)}}{\text{P(not moving)}}$
- Before lab 8 = 0.2*0.96 / 0.51 = 0.38
- After lab 8 = 0.5*0.96 / 0.68 = 0.71





Bayesian Inference

- Bayesian inference = guessing in the style of Bayes

$$P(x|y) = \frac{P(y|x)\,P(x)}{P(y)}$$

likelihood — $P(y|x)$

prior — $P(x)$

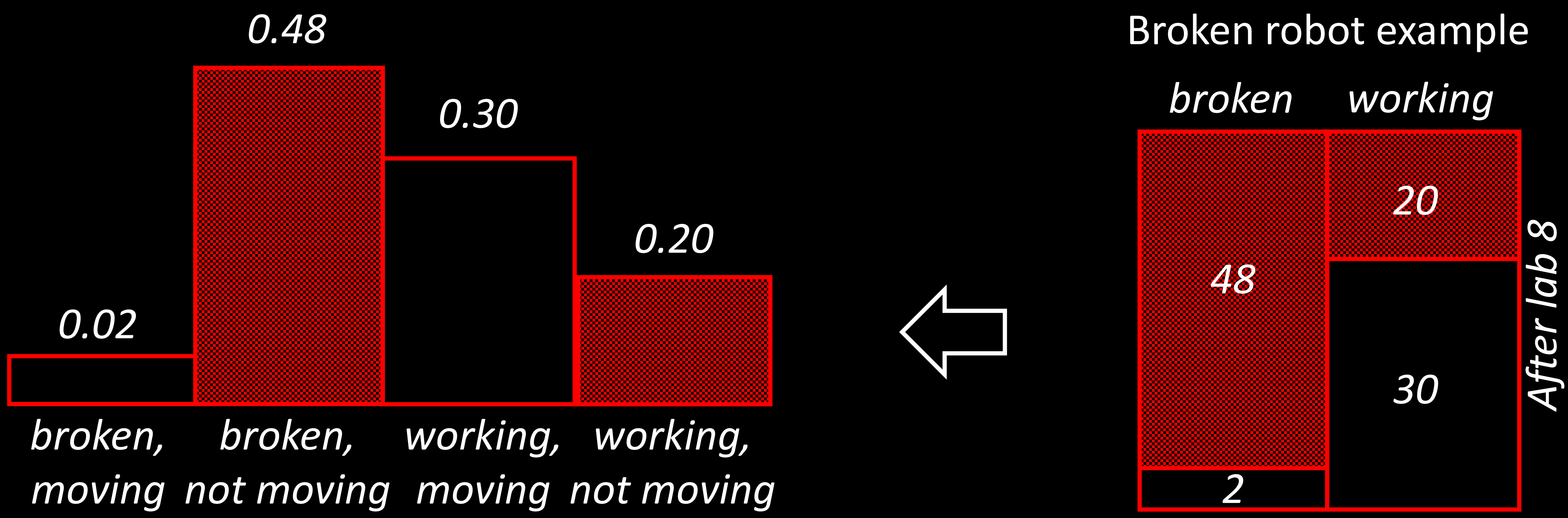
posterior — $P(x|y)$

marginal likelihood (constant) — $P(y)$

- y = Sensor data
- x = Robot state/ location

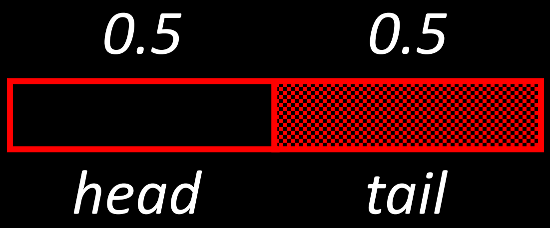
Probability Distribution

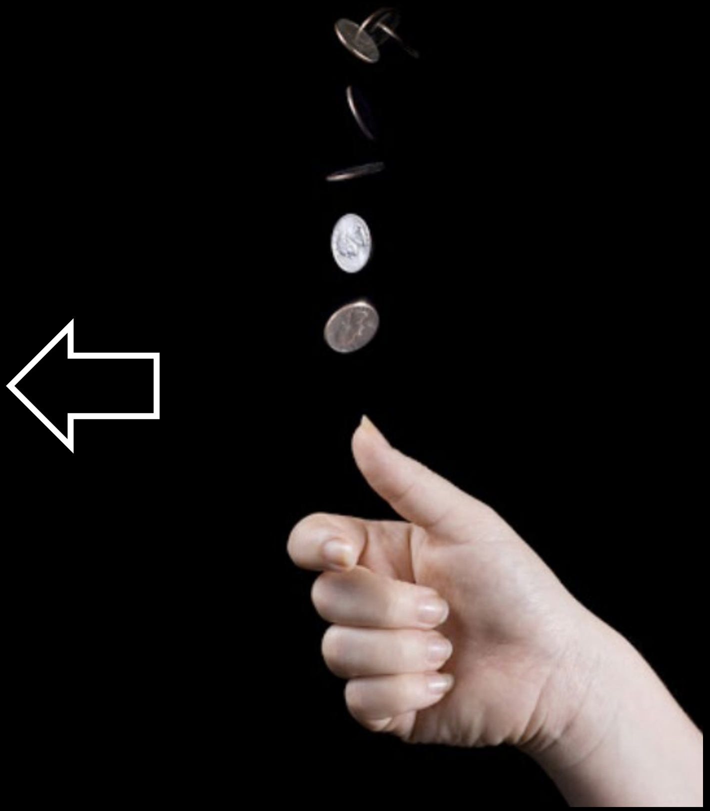
- Beliefs



Probability Distribution

- Beliefs





Probability Distribution

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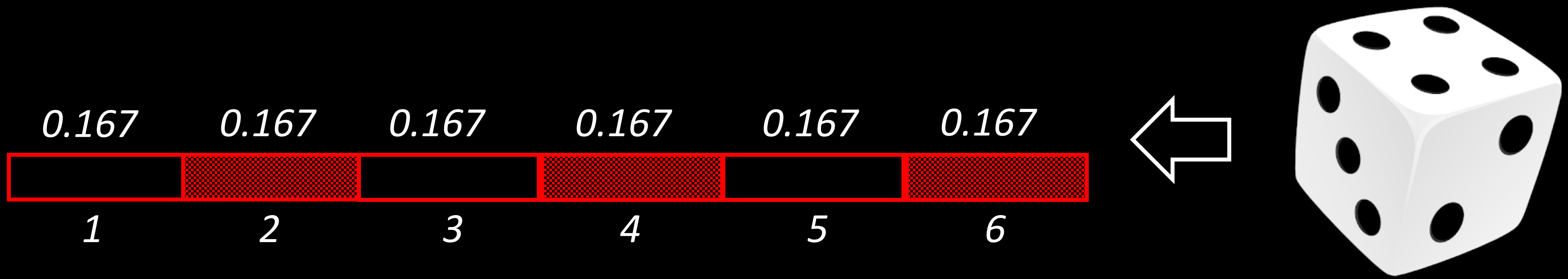
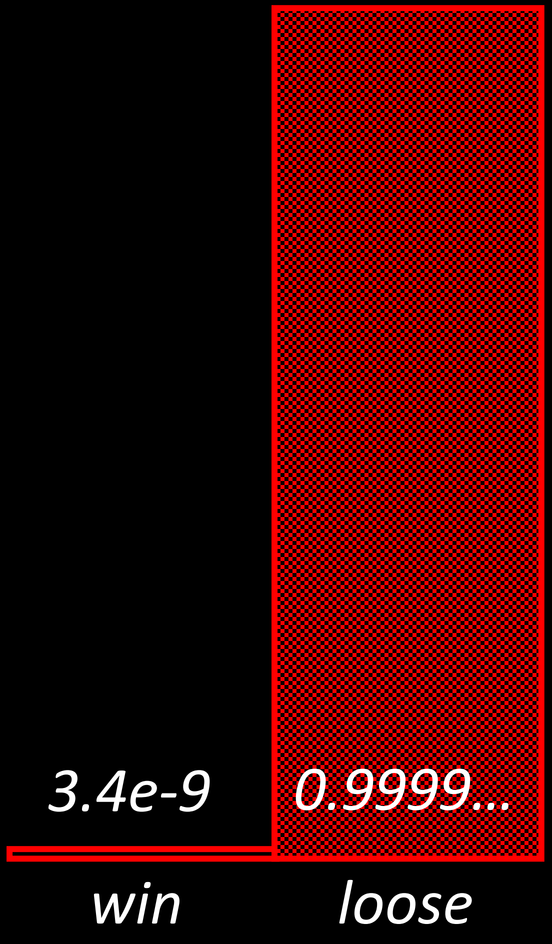


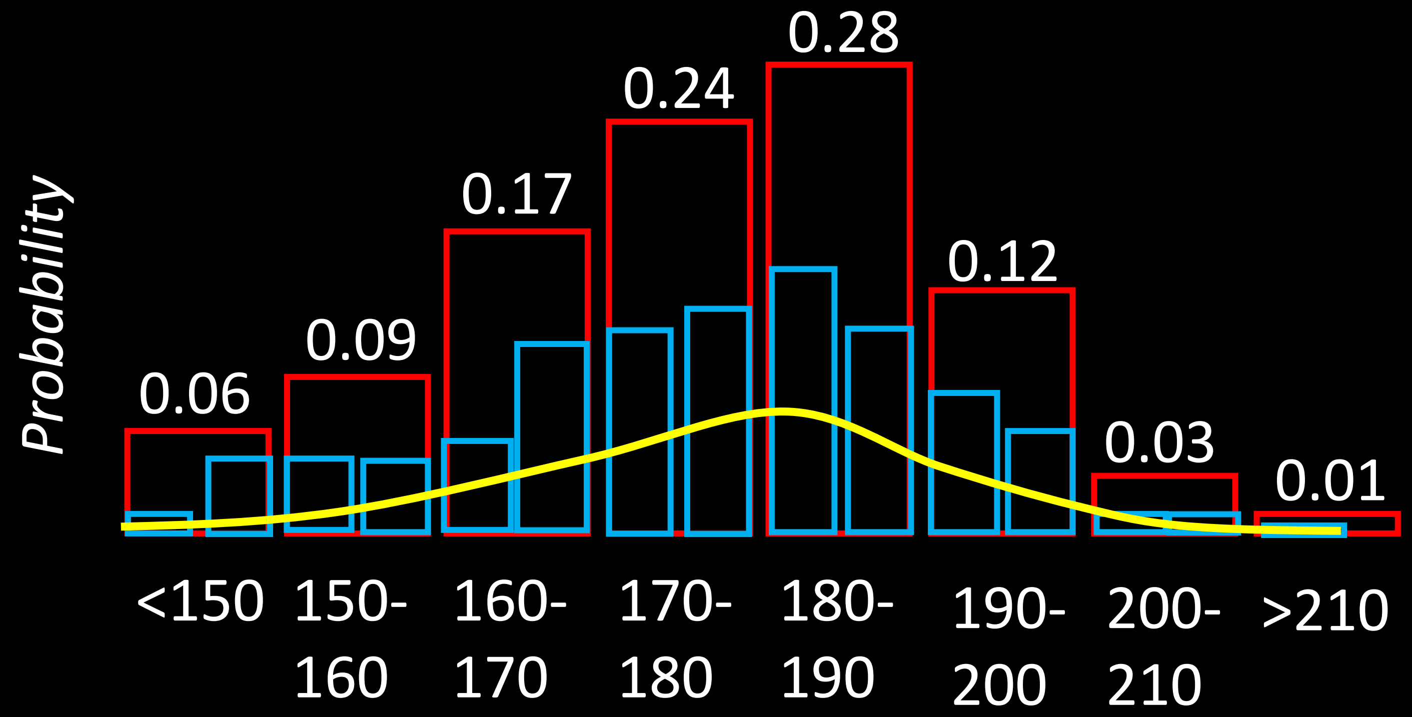
Probability Distribution

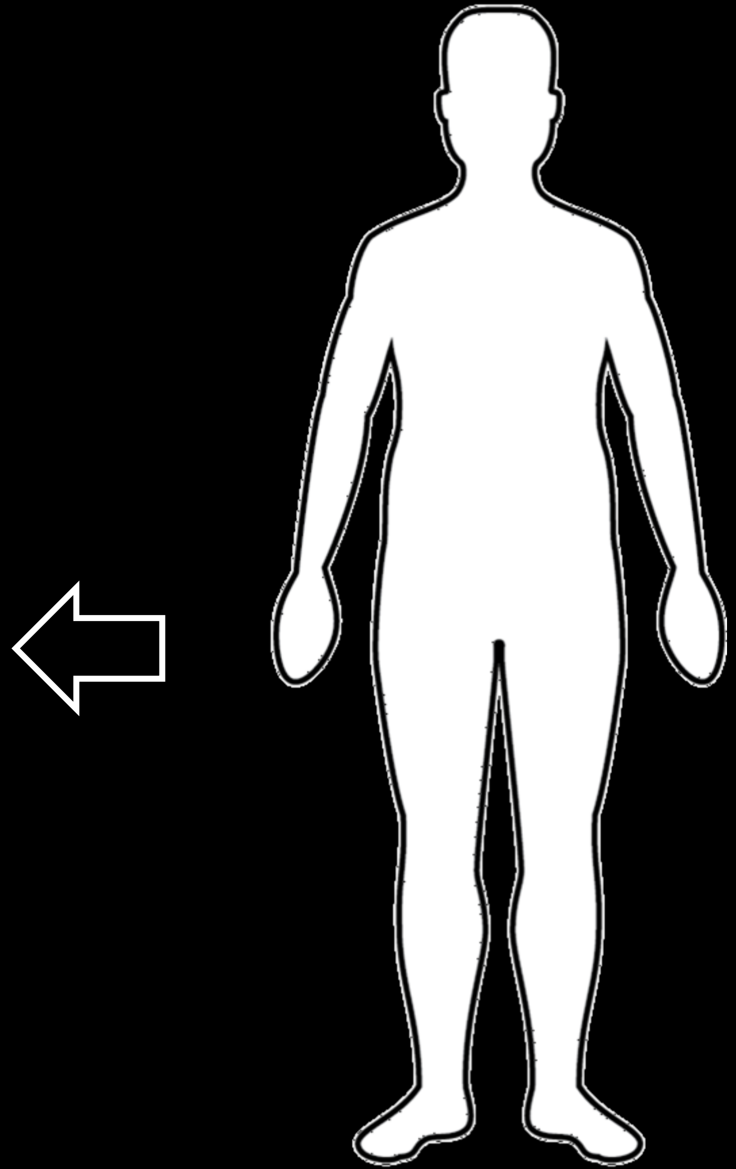
- Beliefs



Probability Distribution

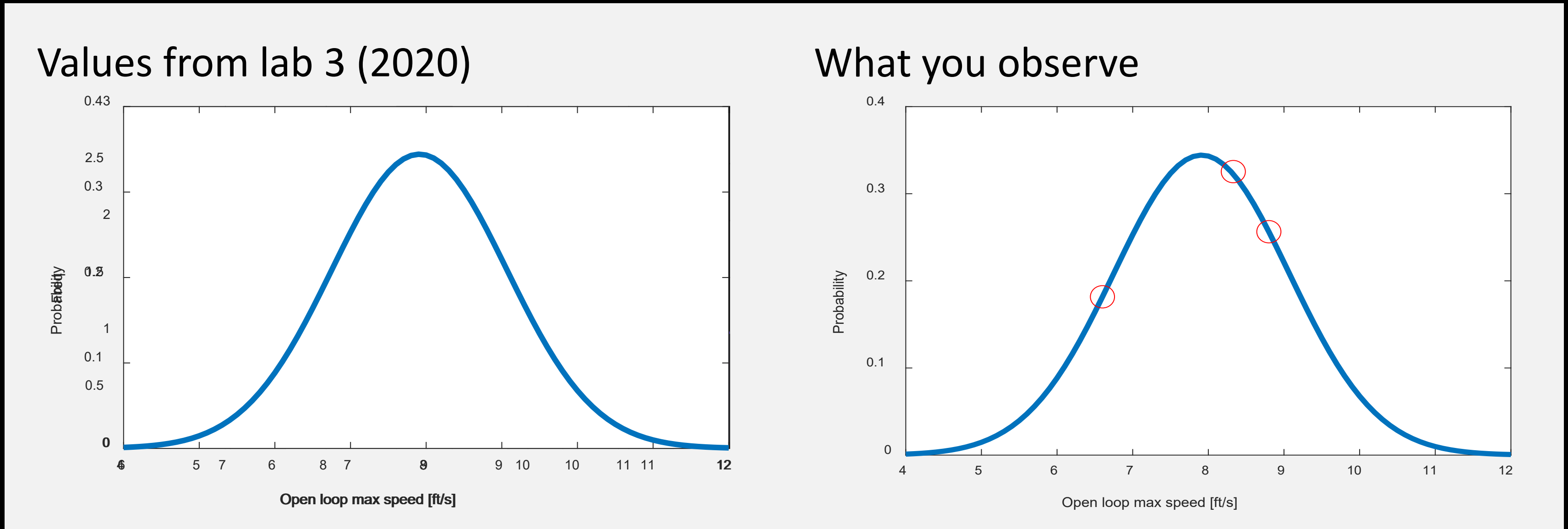
- Beliefs
- Discrete -> continuous *probability distribution*
  - Mean, median, most common value, etc.





Probability Distributions

- What is the maximum speed of your robot?
  - Your speed is 8.8 ft/s, 6.6 ft/s, 8.33 ft/s, but what is the actual value?
- Frequentist Statistics
  - Mean: $\mu = (8.8+6.6+8.33)/3 = 7.91$ ft/s
  - Variance: $\sigma^2 = ((8.8-7.91)^2 + (6.6-7.92)^2 + (8.33-7.91)^2)/(3-1) = 1.35$ ft/s
  - Standard deviation: $\sigma = \text{sqrt}(\sigma^2) = 1.16$ ft/s
  - Standard error: $\sigma / \text{sqrt}(3) = 0.67$ ft/s
- Bayesian Statistics
  - Probably 7.91ft/s…



Probability Distributions

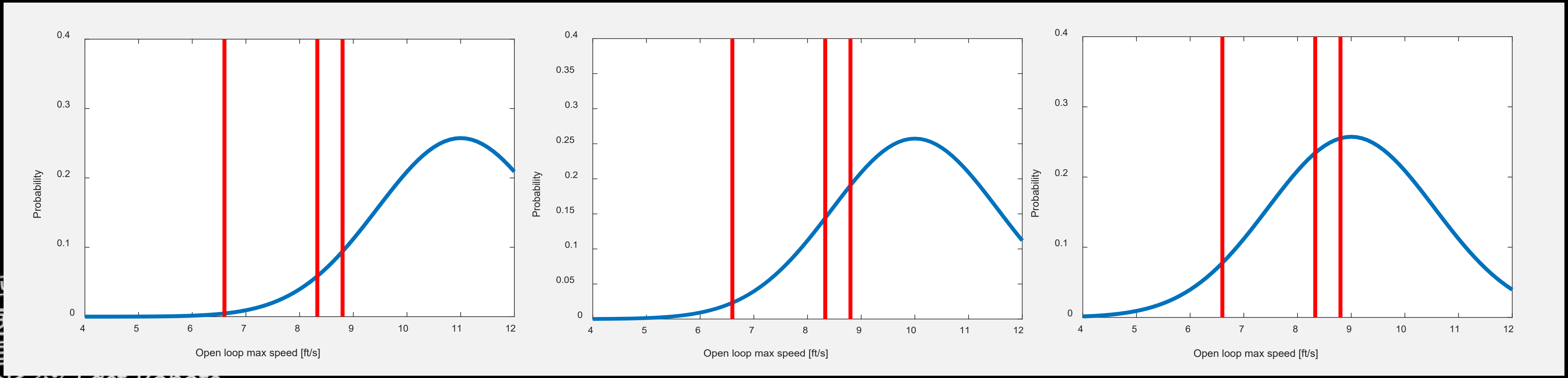
$$P(x|y) = \frac{P(y|x)\, P(x)}{P(y)}$$

- Use Bayes theorem
- Instead of events x and y
  - Substitute “s” for the actual speed
  - Substitute “m” for the measurements
- P(s) is our prior
- P(m|s) is the likelihood associated with those measurements
- P(s|m) is what we believe about the speed given those measurements
- P(m) is the marginal likelihood
- Procedure:
  - Start with a belief
  - Update it
  - End up with a new belief!

Probability Distributions

$$P(x|y) = \frac{P(y|x)\,P(x)}{P(y)}$$

- Use Bayes theorem
- Start by assuming nothing
  - P(s) = uniform
  - $P(s|m) = P(m|s) * c_1/c_2$
  - Simplified: P(s|m) = P(m|s)
    - *Guess!* What if the actual max speed is 11 ft/s?
    - P(s=11|m=[6.6,8.33,8.8]) = P(m=[6.6,8.33,8.8] | s=11)
    - P(m = 6.6 | s = 11) * P(m = 8.33 | s = 11) * P(m = 8.8 | s = 11)



Probability Distributions

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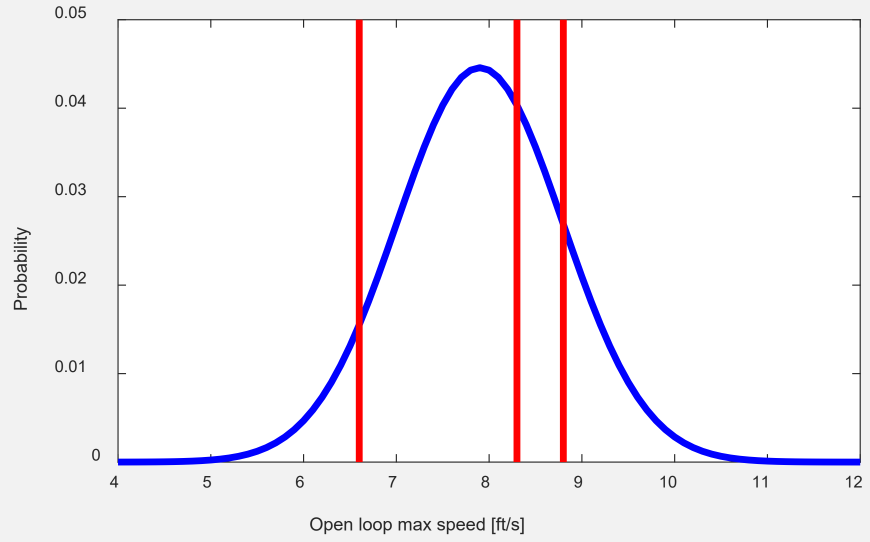


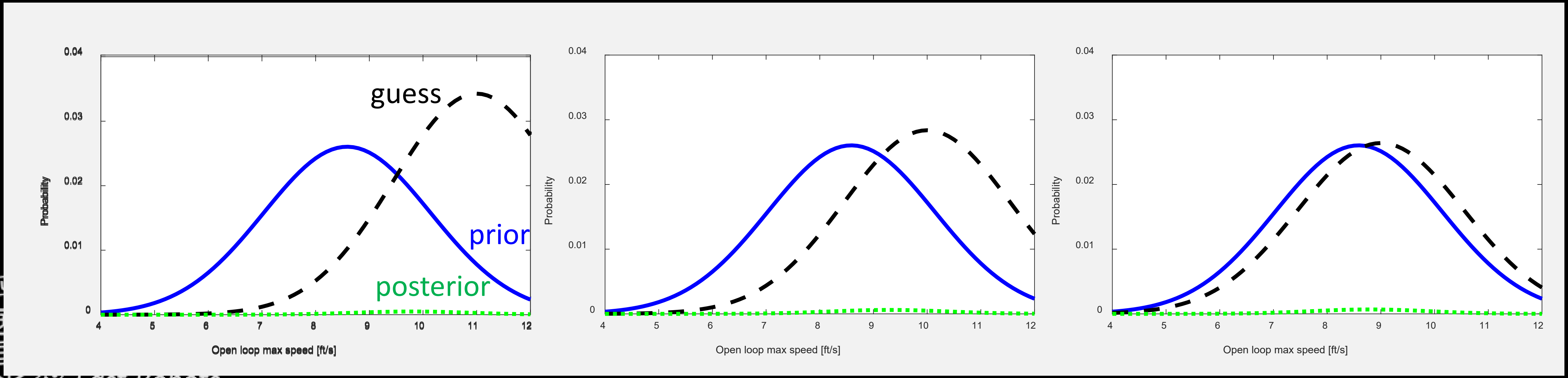
No prior:
Maximum Likelihood Estimate
(MLE)

Probability Distributions

$$P(x|y) = \frac{P(y|x)\,P(x)}{P(y)}$$

- Use Bayes theorem
- Add a prior!
  - You know yesterday's speed, and you can kind of judge the current speed by eye
    - Prior: 7.91 ft/s $\pm$ 1.16ft/s
  - P(s = 11 | m = [6.6, 8.33, 8.8]) = P(m = [6.6, 8.33, 8.8] | s = 11) * P(s = 11)
    = P(m=6.6|s=11)*P(s=11) * P(m=8.33|s=11)*P(s=11) * P(m=8.8|s=11)*P(s=11)

Repeat the process!



Probability Distributions

$$P(x|y) = \frac{P(y|x)\,P(x)}{P(y)}$$

- Use Bayes theorem
- Add a prior!
  - You know yesterday’s speed, and you can kind of judge the current speed by eye
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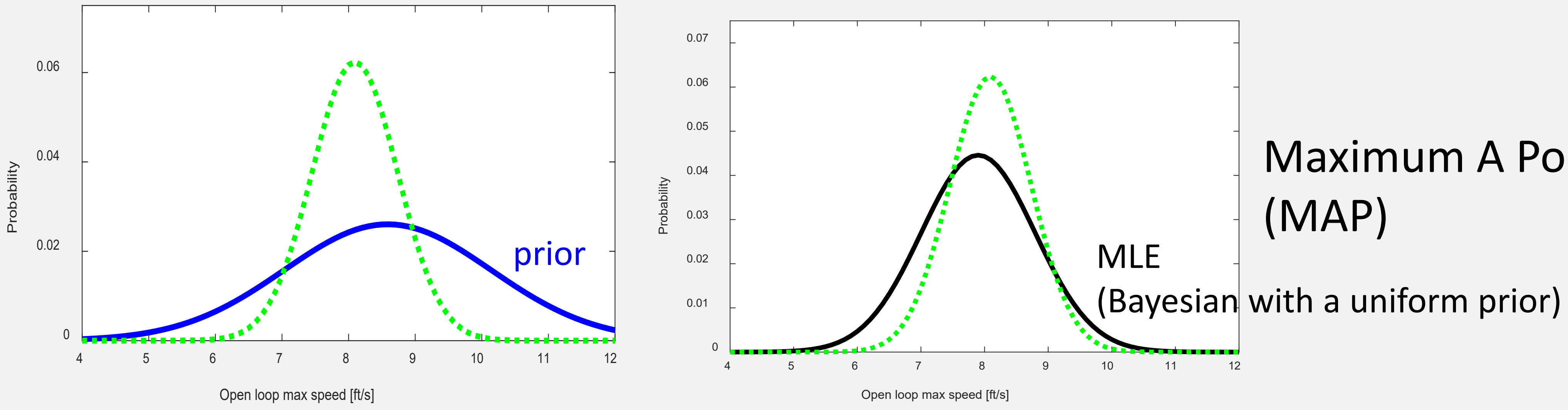
    = P(m=6.6|s=11)*P(s=11) * P(m=8.33|s=11)*P(s=11) * P(m=8.8|s=11)*P(s=11)

Repeat the process!

Add everything up to get the posterior distribution



Maximum A Posteriori (MAP)

Bayesian Inference

likelihood

prior

$$P(x|y) = \frac{P(y|x)\,P(x)}{P(y)}$$

conditional probability
posterior

marginal likelihood
(constant)

Probability Distributions

- Always believe the impossible, at least a little bit!
- Leave room for believing the unlikely. Leave a non-zero probability unless you are *absolutely* certain.
- “It ain’t what you don’t know that gets you into trouble. It’s what you know for sure that just ain’t so.” –Mark Twain
- “Alice laughed “there’s no use trying”, she said: “one can’t believe impossible things. “I daresay you haven’t had much practice.” said the Queen. “When I was younger, I always did it for half an hour a day. Why sometimes, I’ve believed as many as six impossible things before breakfast.”

Alice’s adventures in wonderland

Probabilistic Robotics

+ Explicitly represent the uncertainty using probability theory
+ Accommodate inaccurate models
+ Accommodate imperfect sensors
+ Robust in real-world applications
+ Best known approach to many hard robotics problems

- Computationally demanding
- Need to approximate
- False assumptions

Is Robotics Going Statistics?
The Field of Probabilistic Robotics

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Carnegie Mellon University
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draft, please do not circulate

Abstract

In the 1970s, most research in robotics presupposed the availability of exact models, of robots and their environments. Little emphasis was placed on sensing and the intrinsic limitations of modeling complex physical phenomena. This changed in the mid-1980s, when the paradigm shifted towards reactive techniques. Reactive controllers rely on capable sensors to generate robot control. Rejections of models were typical for researchers in this field. Since the mid-1990s, a new approach has begun to emerge: probabilistic robotics. This approach relies on statistical techniques to seamlessly integrate imperfect models and imperfect sensing. The present article describes the basics of probabilistic robotics and highlights some of its recent successes.

References

- Probabilistic Robotics, book by *Dieter Fox, Sebastian Thrun, and Wolfram Burgard*
- How Bayes Theorem works (Youtube), by Brandon Rohrer