Dr. Jonathan Jaramillo
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ECE 4160/5160
MAE 4910/5910

Fast Robots

Observability

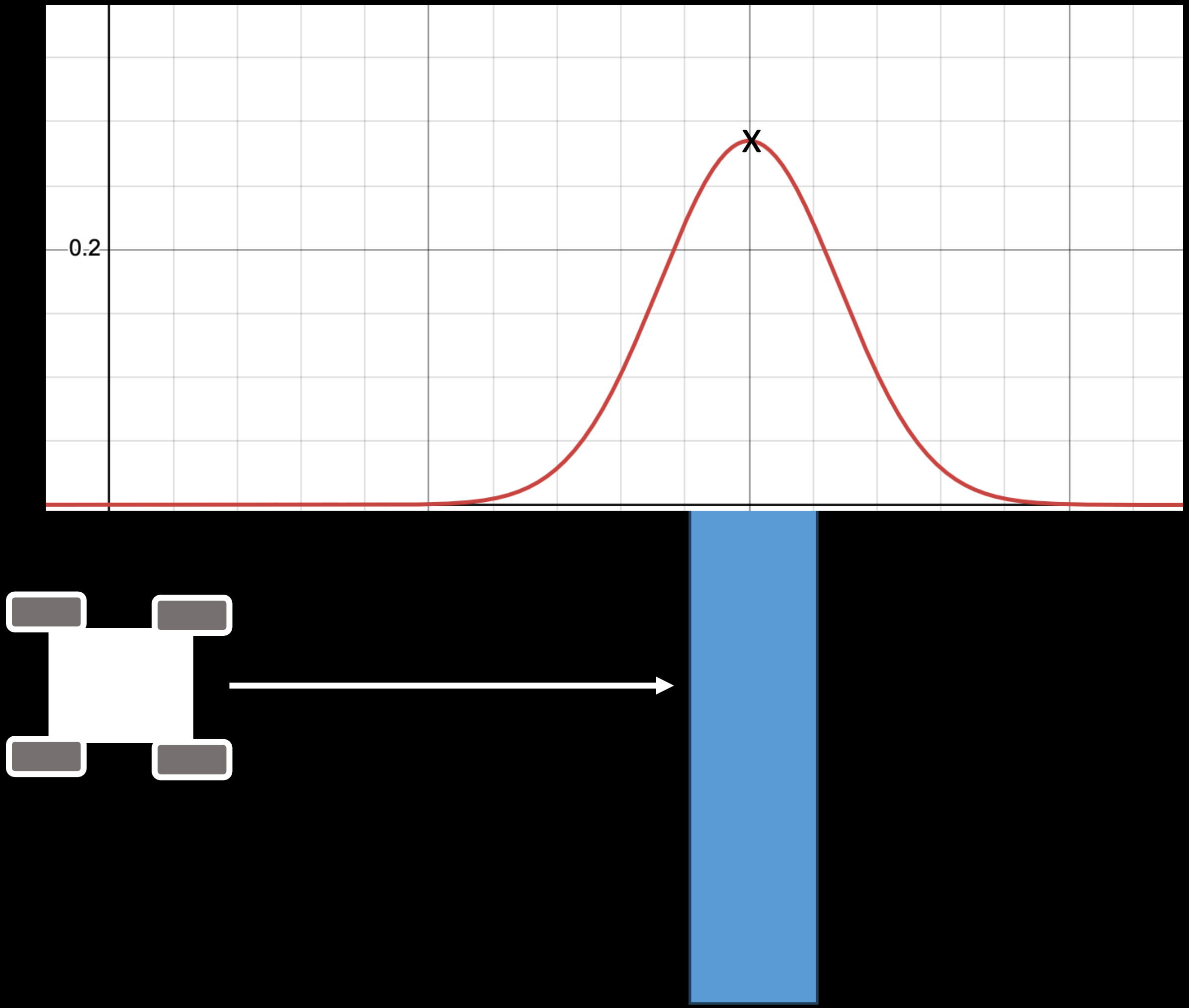
- Bayesian inference = guessing in the style of Bayes

$$\underbrace{P(x|y)}_{\text{posterior}} = \frac{\overbrace{P(y|x)}^{\text{likelihood}}\,\overbrace{P(x)}^{\text{prior}}}{\underbrace{P(y)}_{\text{marginal likelihood (constant)}}}$$

- y = Sensor data
- x = Robot state/ location

Example

- $p(x|y) = p(y|x)$
- $y = x + N(0,1.4)$
- $p(x = 10|y = 10) = p(y = 10|x = 10)$
  $= 0.285$
- $p(x = 8|y = 10) = p(y = 10|x = 8)$
  $= 0.103$



$$P(x|y) = \frac{P(y|x)P(x)}{P(y)}$$

Example

- $p(x|y) = p(y|x)$
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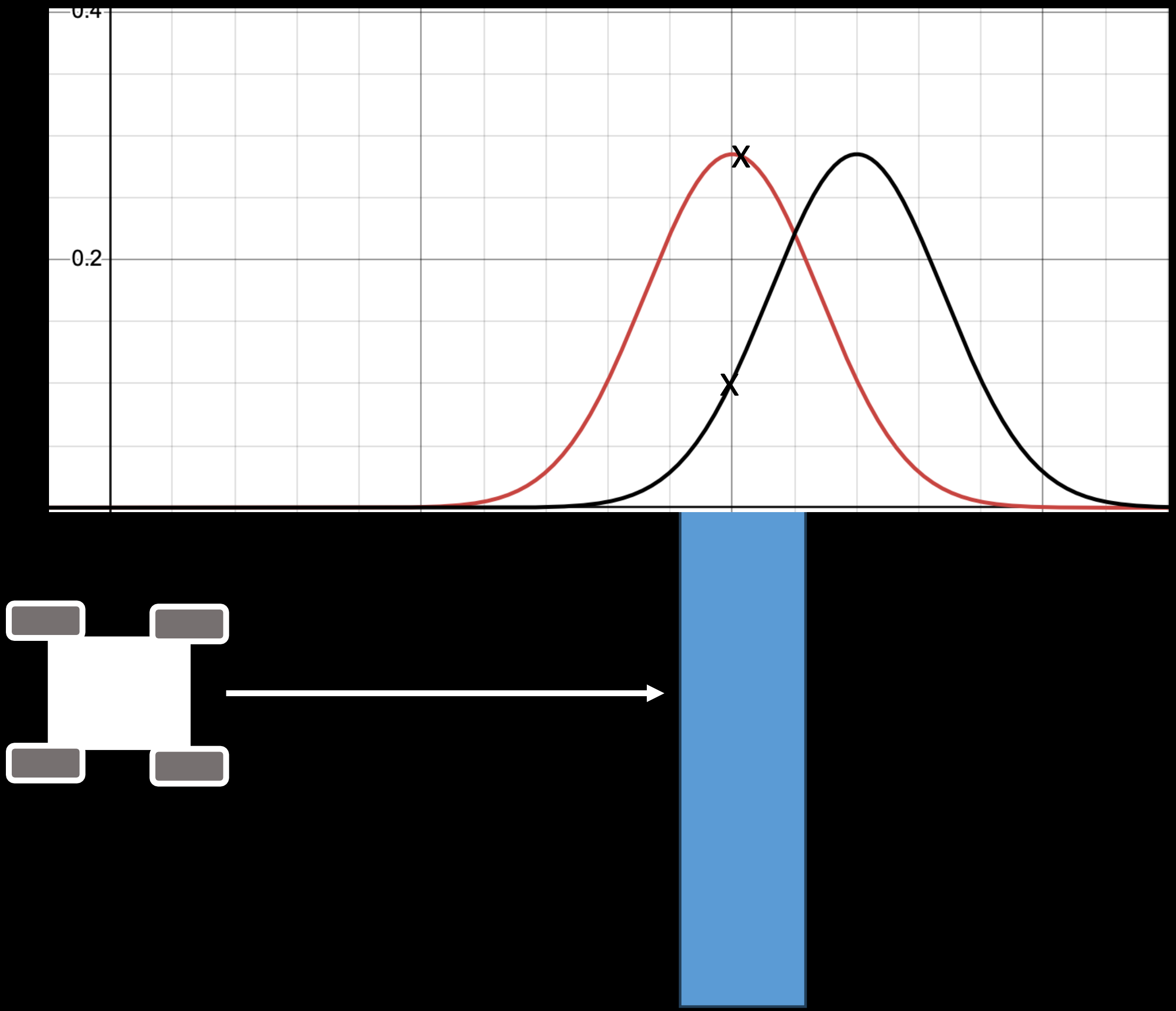
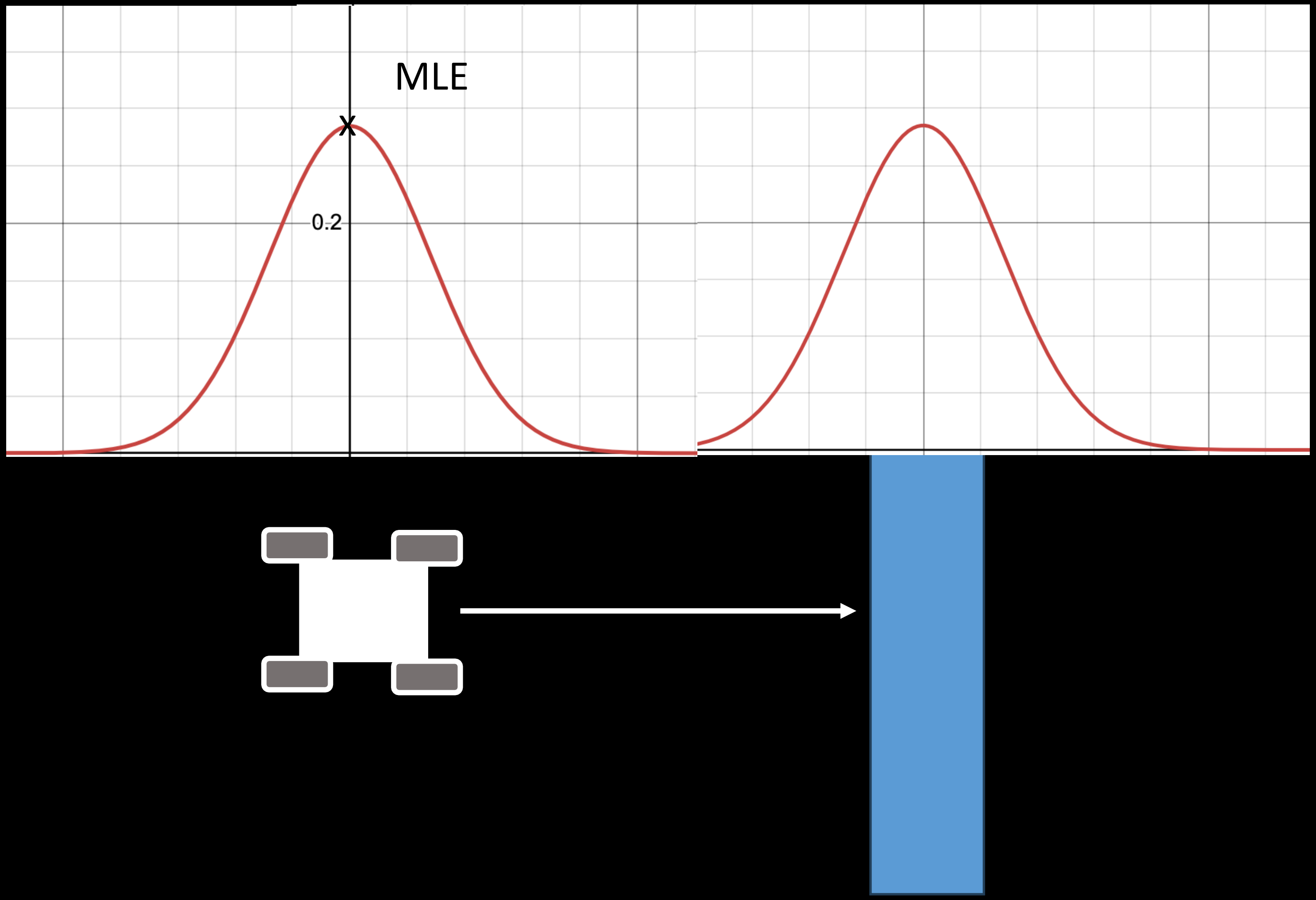


$$P(x|y) = \frac{P(y|x)P(x)}{P(y)}$$

Example

- $p(x|y) = p(y|x)$
- $y = x + N(0,1.4)$
- $p(x = 10|y = 10) = 0.285$
- $p(x = 8|y = 10) = 0.103$
- $p(x|y) = N(y, 1.4)$



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Example

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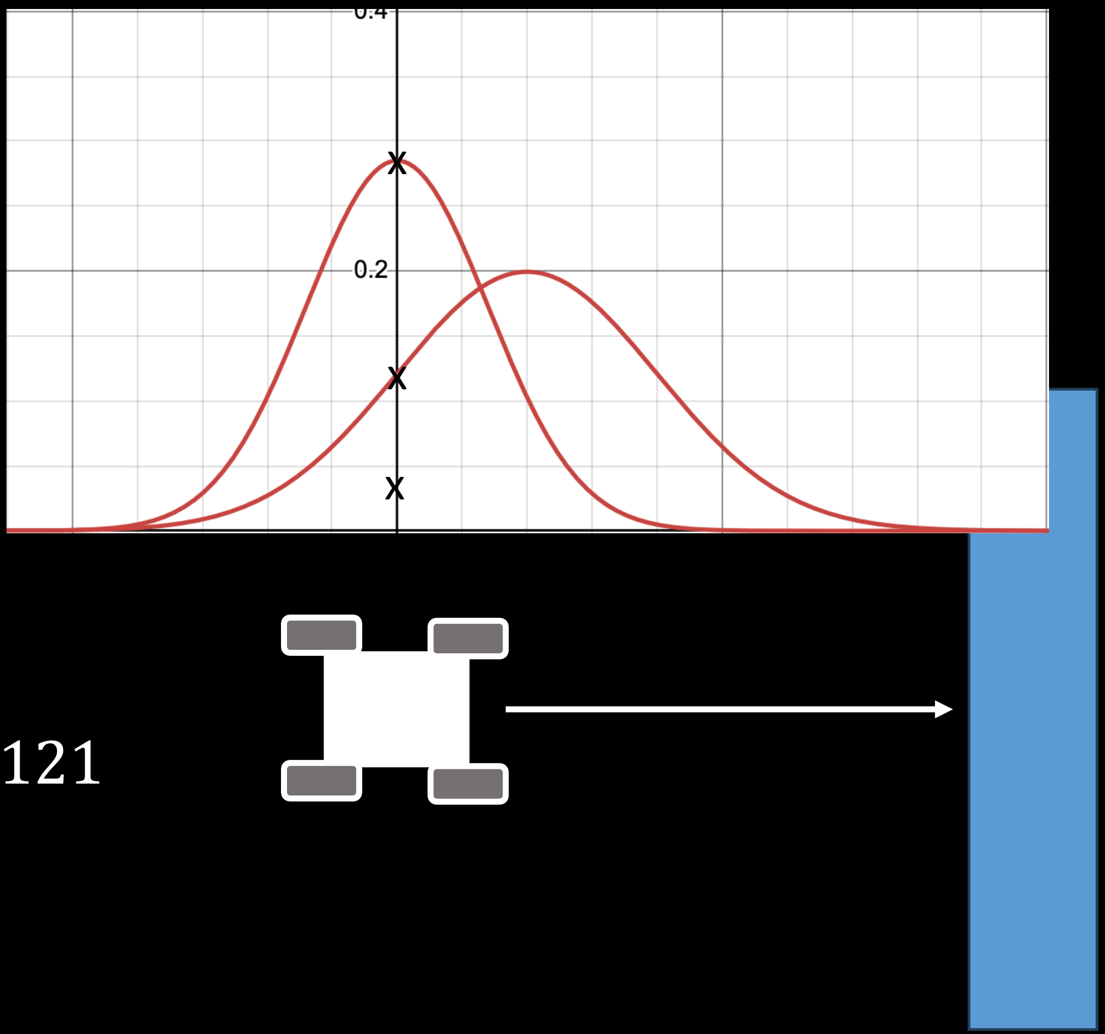
- $p(x = 10|y = 10) = 0.285$
- $p(x = 8|y = 10) = 0.103$
- $p(x|y) = N(0,1.4)$
- Prior – $p(x) = N(2,2)$
- $p(x|y) = p(y|x)p(x)$
- $p(x = 10|y = 10) = 0.285 \times 0.121$
  $= .034$



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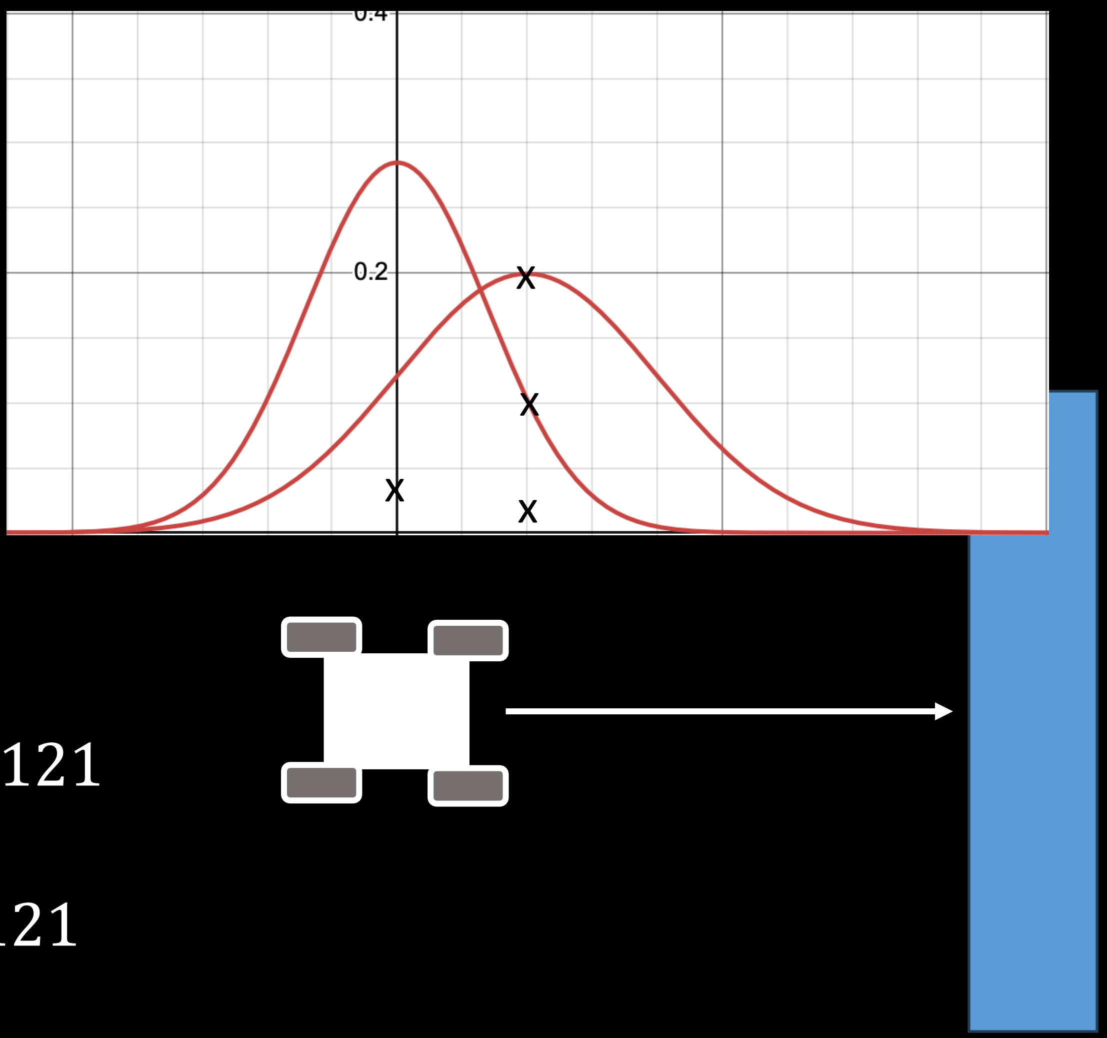

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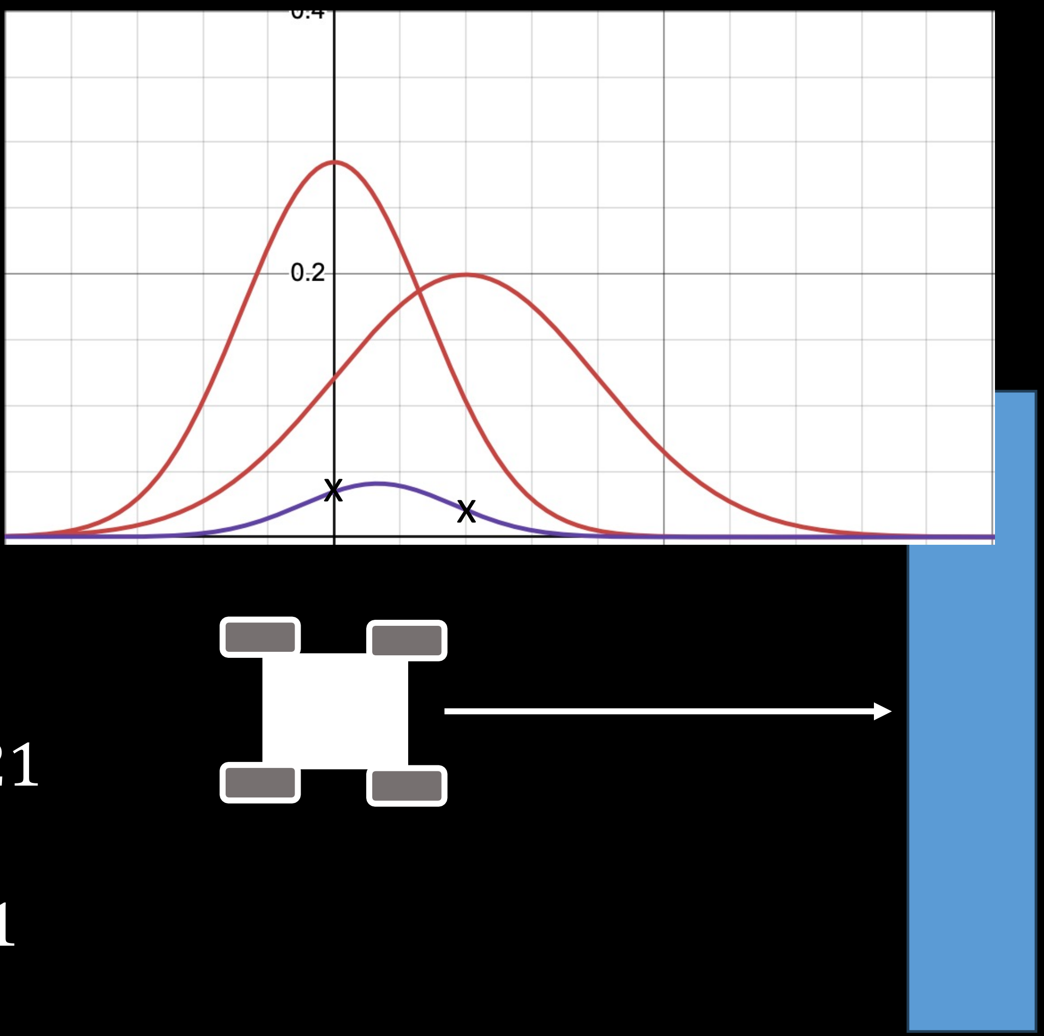

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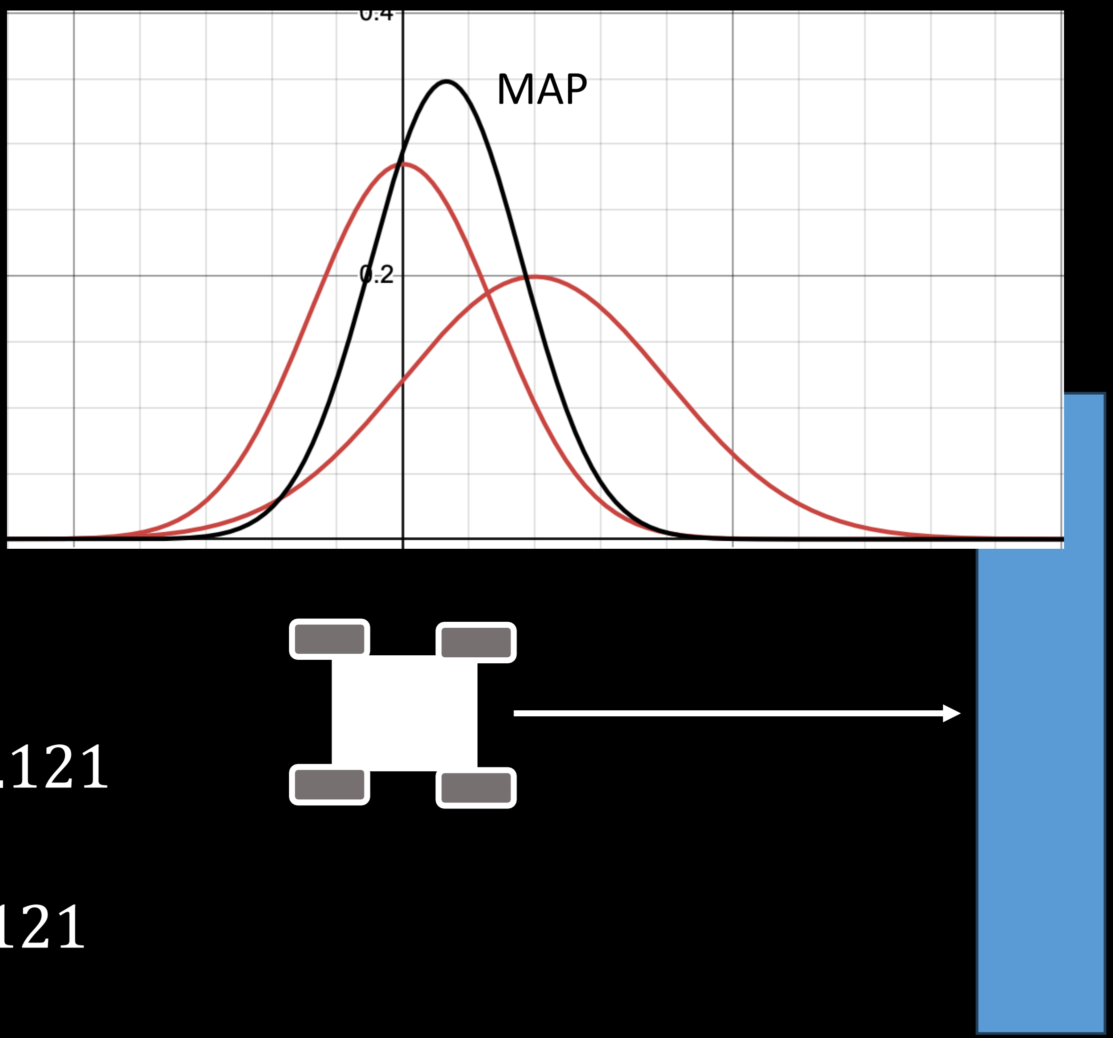


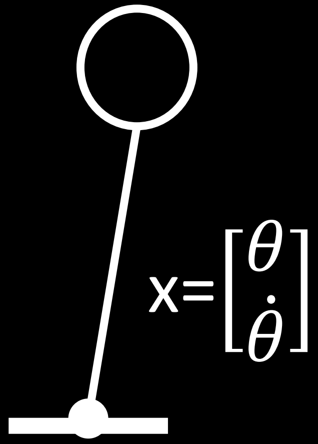
$$P(x|y) = \frac{P(y|x)P(x)}{P(y)}$$

Based on “Control Bootcamp”, Steve Brunton, UW
https://www.youtube.com/watch?v=Pi7l8mMjYVE

Linear Systems

- Linear systems review
- Eigenvectors and eigenvalues
- Stability
- Discrete time systems
- Linearizing non-linear systems
- Controllability
- LQR
- Observability

$$\dot{x} = Ax + Bu$$



This should look familiar from..
- MATH 2940 Linear Algebra
- ECE3250 Signals and systems
- ECE5210 Theory of linear systems
- MAE3260 System Dynamics
- etc…

Linear Systems Control – "review of review"

- Linear system: $\dot{x} = Ax$
- Solution: $x(t) = e^{At}x(0)$
- Eigenvectors: $T = [\xi_1 \quad \xi_2 \quad \dots \quad \xi_n]$
- Eigenvalues: $\mathrm{D} = \begin{bmatrix} \lambda_1 & & & 0 \\ & \lambda_2 & & \\ & & \dots & \\ 0 & & & \lambda_n \end{bmatrix}$
    - `>>[T,D] = eig(A)`
- Linear transform: $AT = TD$
- Solution: $e^{At} = Te^{Dt}T^{-1}$
- Mapping from x to z to x: $x(t) = Te^{Dt}T^{-1}x(0)$
- Stability in continuous time: $\lambda = a + ib$, stable iff a<0
    - Discrete time: $x(k+1) = \tilde{A}x(k), \tilde{A} = e^{A\Delta t}$
        - Stability in discrete time: $\tilde{\lambda}^n = R^n e^{in\theta}$, stable iff R<1

- Linearizing non-linear systems
    - Fixed points
    - Jacobian
- Controllability
    - $\dot{x} = (A - BK)x$
    - `>>rank(ctrb(A,B))`
- Reachability
- Controllability Gramian
- Pole placement
    - `>>K=place(A,B,p)`
- Optimal control (LQR)
    - `>>K=lqr(A,B,Q,R)`

Linear Quadratic Control

$$\dot{x} = Ax + Bu, x \epsilon \mathbb{R}^n$$
$$u = -Kx$$
$$\dot{x} = (A - BK)x$$

- `>> K = place(A,B,eigs)`
- Where are the best eigs??
  - Linear Quadratic Regulator (LQR)
    - `>> K = lqr(A,B,Q,R)`
    - Riccati equation
      - $\int_0^\infty (x^T Q x + u^T R u) dt$
    - $Q = \begin{bmatrix} 1 & & & 0 \\ & 1 & & \\ & & 10 & \\ 0 & & & 100 \end{bmatrix}, R = 0.01$

Linear Systems

Based on “Control Bootcamp”, Steve Brunton, UW
https://www.youtube.com/watch?v=Pi7l8mMjYVE

- Linear systems review
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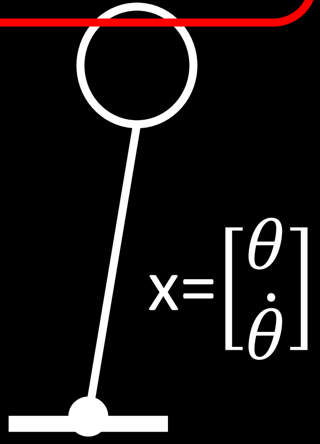
- LQR
- Observability

$$\dot{x} = Ax + Bu$$



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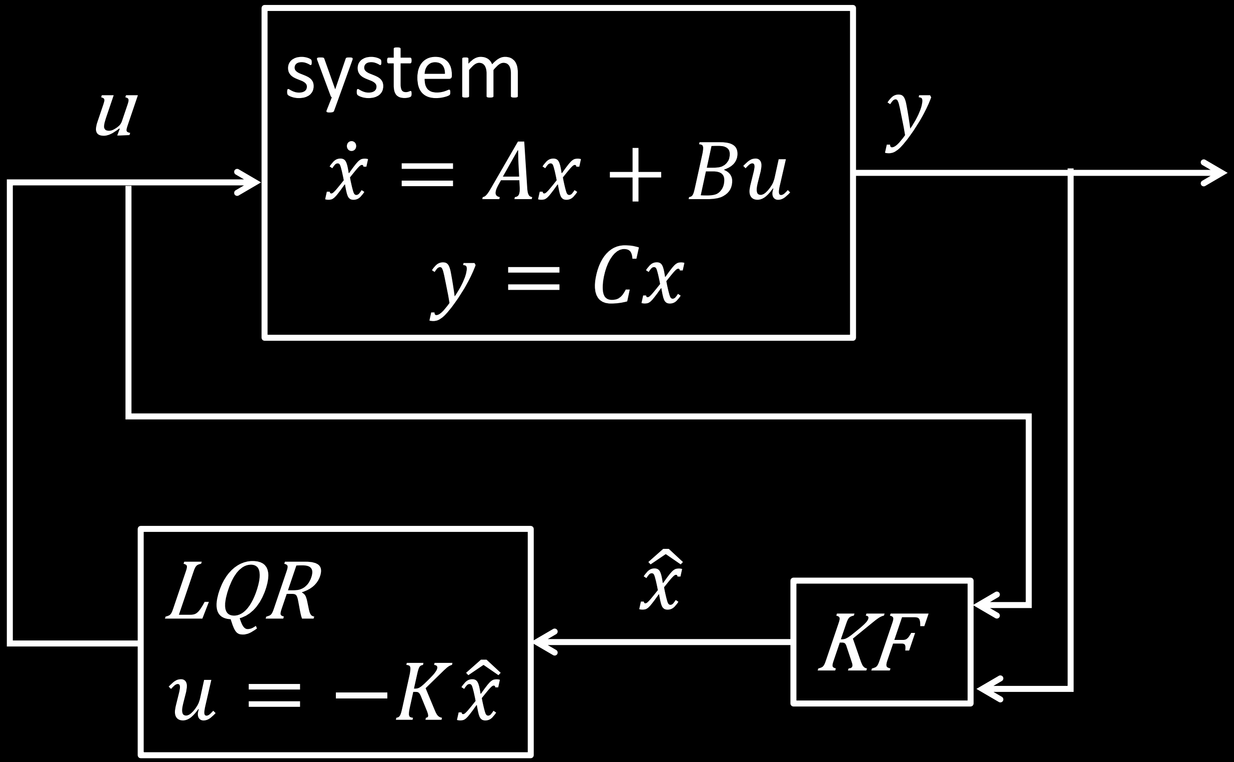
Fast Robots

Observability

Observability

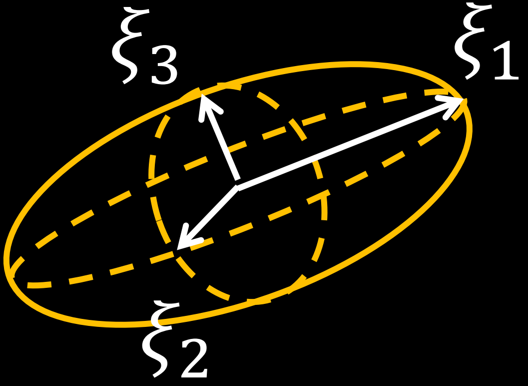
$$\dot{x} = Ax + Bu, x \epsilon \mathbb{R}^n$$
$$u = -Kx$$
$$\dot{x} = (A - BK)x$$

- Controllability
  - Can we steer the system anywhere given some control input u?
- Observability
  - Can we estimate any state x, from a time series of measurements y(t)?



Observability

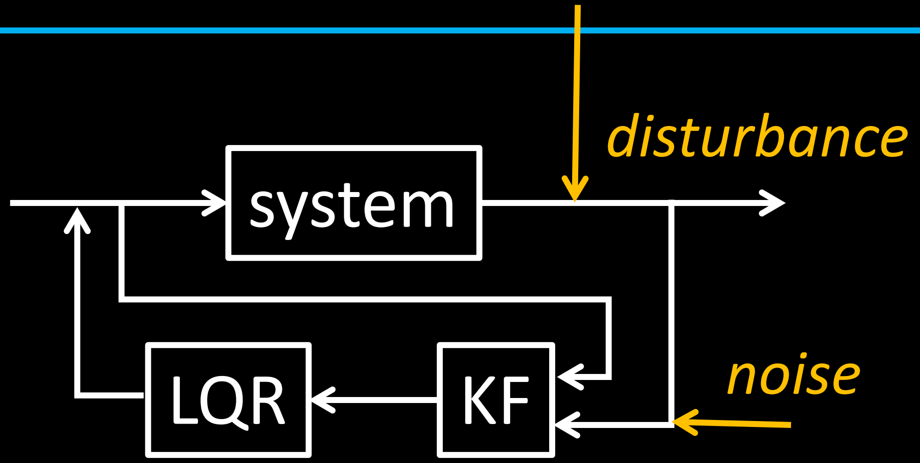
- $\mathcal{O} = \begin{bmatrix} C \\ CA \\ CA^2 \\ \dots \\ CA^{n-1} \end{bmatrix}$



1. Observable iff rank($\mathcal{O}$) = n
   - `>>rank(obsv(A,C))`
2. Iff a system Is observable, we can estimate x from y
   - Observability Gramian
     - `>>[U,Σ, V]=svd(`$\mathcal{O}$`)`

$$\dot{x} = Ax + Bu \ +d \qquad x\epsilon\mathbb{R}^n$$
$$y = Cx \ +n \qquad u\epsilon\mathbb{R}^q$$
$$y\epsilon\mathbb{R}^p$$

- Controllability
- $\mathbb{C} = [B \quad AB \quad A^2B \quad \dots \quad A^{n-1}B]$
- >>ctrb(A,B)
- Reachability



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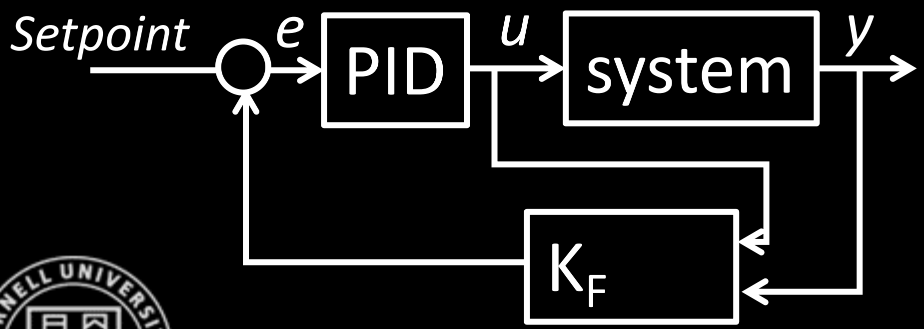
Kalman Filter

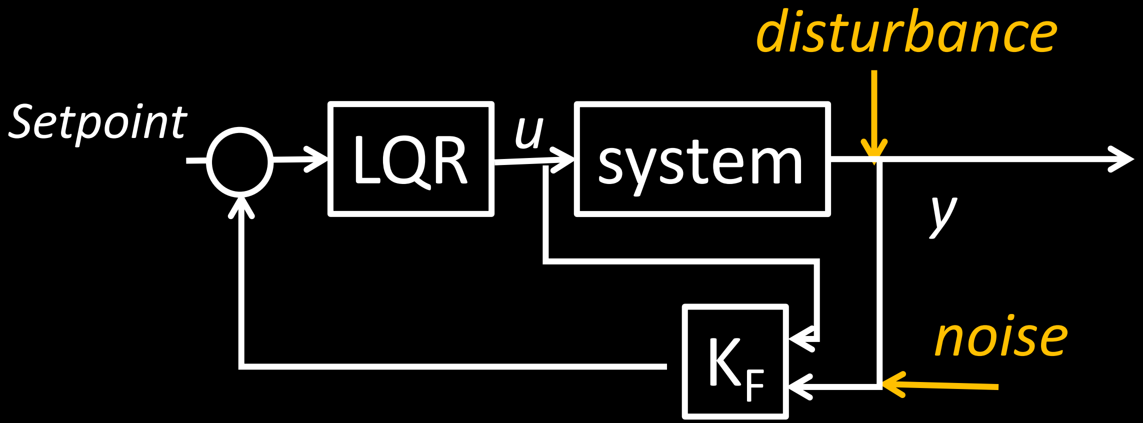
Why sensor fusion?

- Not full state feedback
- Bad sensors
- Imperfect model
- Slow feedback

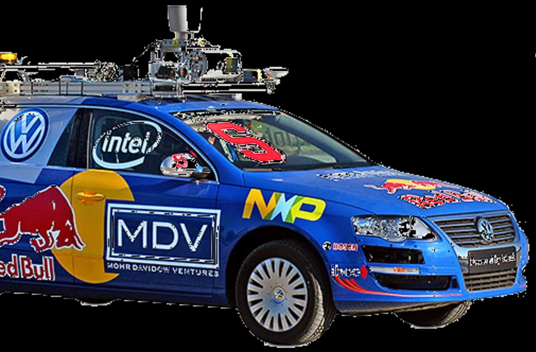
KF with PID



What you typically apply KF on

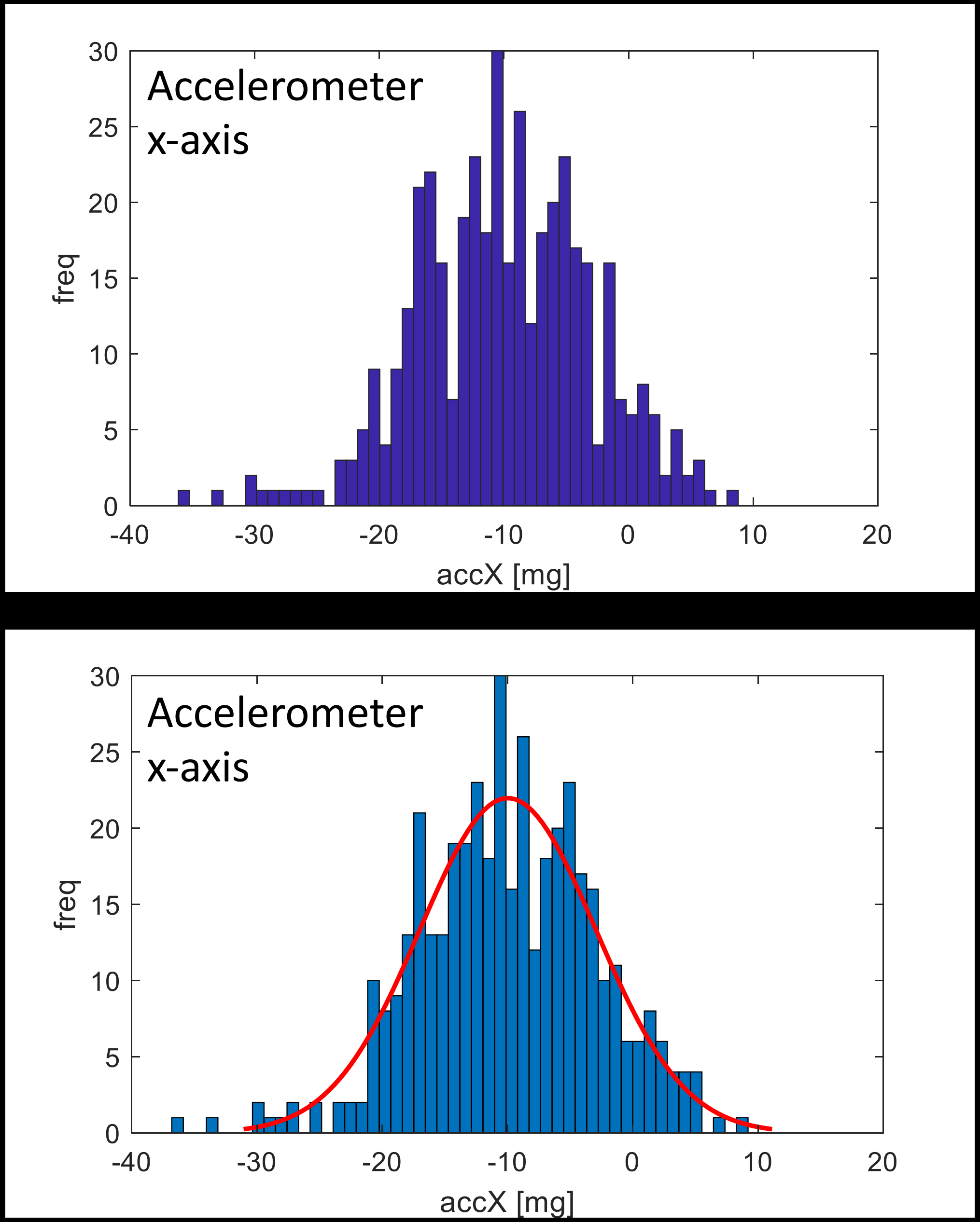


Probabilistic Robotics



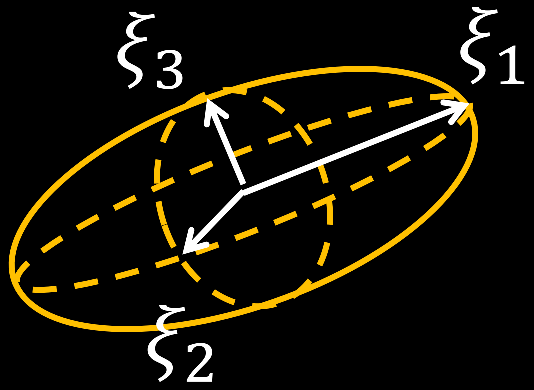
Sources of uncertainty
- Measurements are uncertain
- Actions are uncertain
- Models are uncertain
- States are uncertain

- Gaussian distributions
    - $[\mu \mp \sigma]$
    - Symmetric
    - Unimodal
    - Sum to “unity”



Observability

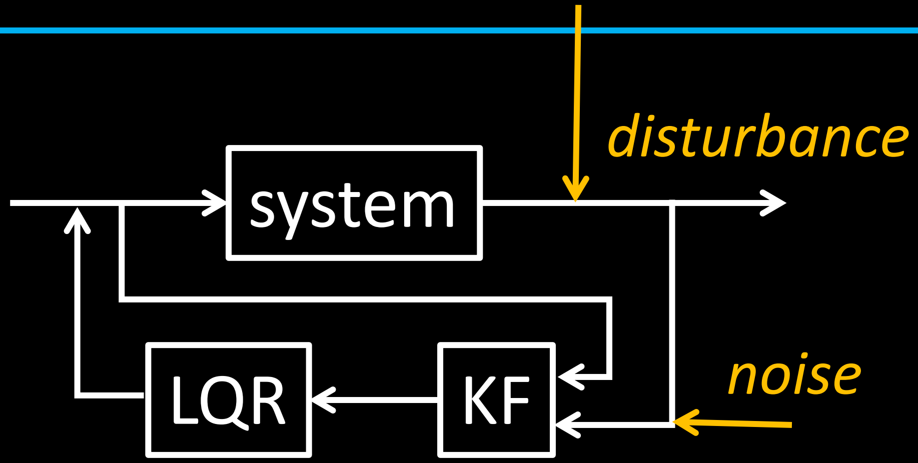
- $\sigma = \begin{bmatrix} C \\ CA \\ CA^2 \\ \dots \\ CA^{n-1} \end{bmatrix}$



1. Observable iff rank(σ) = n
   - `>>rank(obsv(A,C))`
2. Iff a system Is observable, we can estimate x from y
   - Observability Gramian
     - `>>[U,Σ, V]=svd(σ)`

$$\dot{x} = Ax + Bu + d \qquad x \epsilon \mathbb{R}^n$$
$$y = Cx + n \qquad u \epsilon \mathbb{R}^q$$
$$y \epsilon \mathbb{R}^p$$

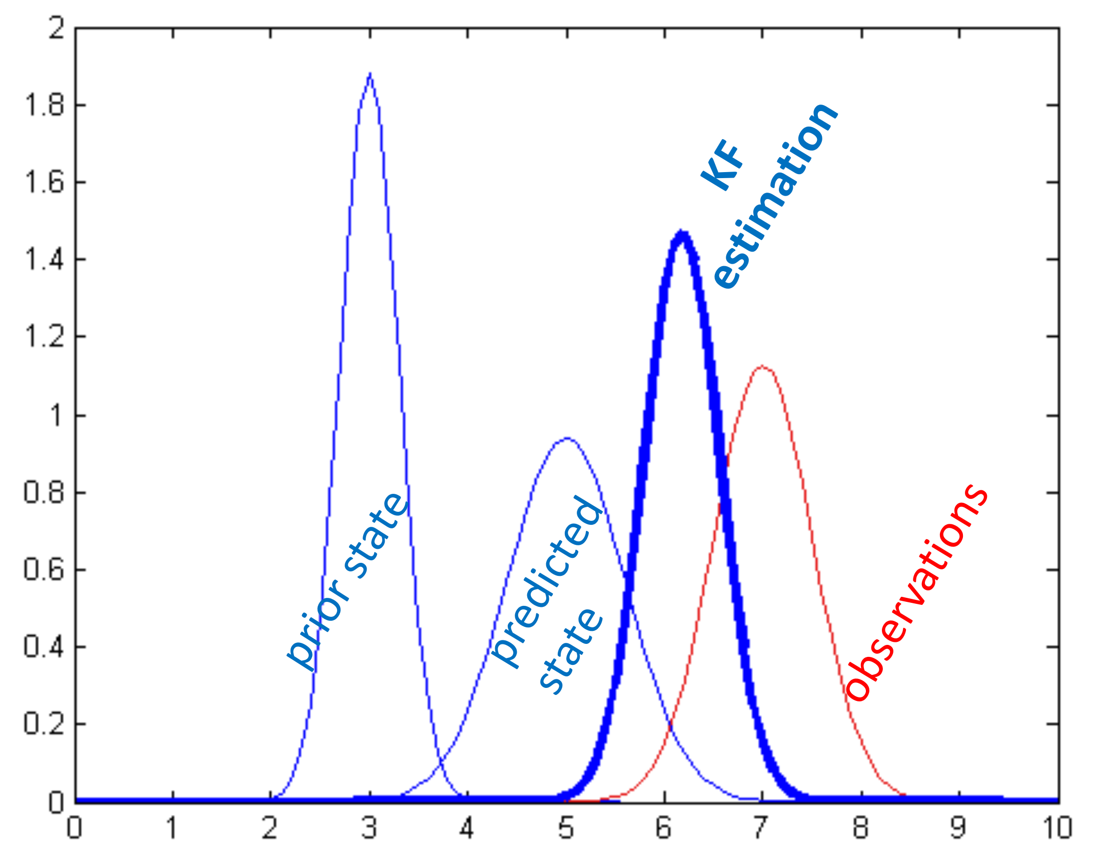
- Controllability
- $\mathbb{C} = [B \quad AB \quad A^2B \quad \dots \quad A^{n-1}B]$
- >>ctrb(A,B)
- Reachability



Kalman Filter

Incorporate uncertainty to get better estimates based on both inputs and observations

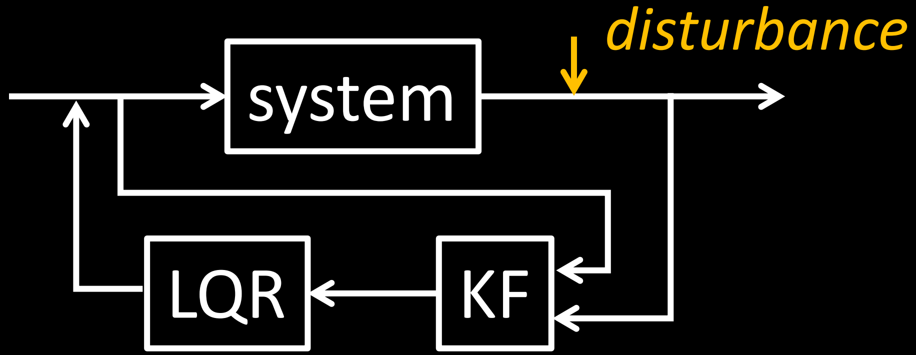
- Assume that posterior and prior belief are Gaussian variables

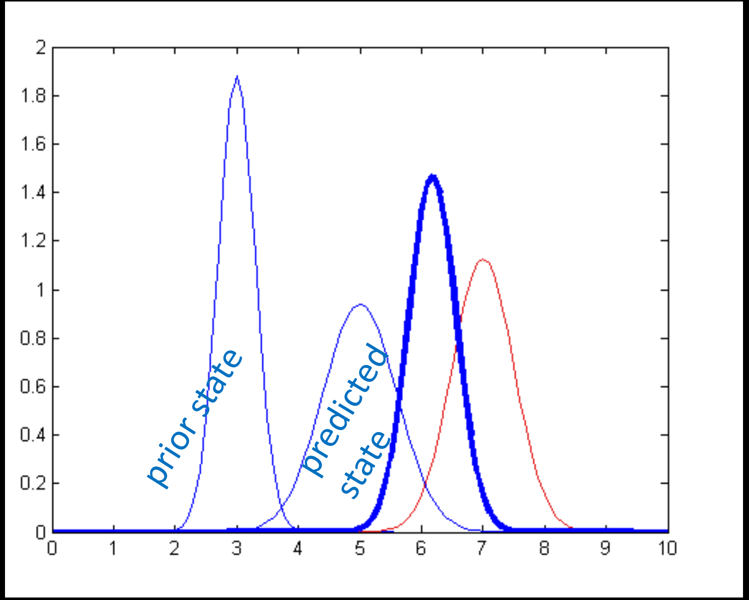


Kalman Filter

- Assume that posterior and prior belief are Gaussian variables
  - Prediction step
    - $x(t) = A\,x(t-1) + Bu(t) + n$, where…
      - $\mu_p(t) = A\,\mu(t-1) + B\,u(t)$
      - $\Sigma_p(t) = A\,\Sigma(t-1)\,A^T + \Sigma_u$
  - Update step

State estimate: $\mu(t)$
State uncertainty: $\Sigma(t)$
Process noise: Σ_u

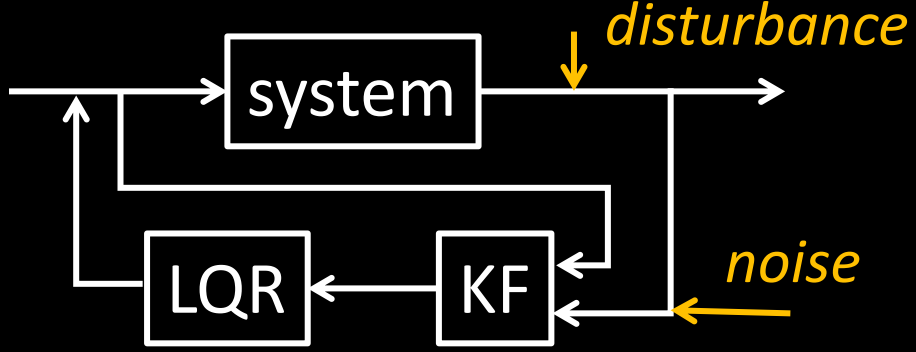


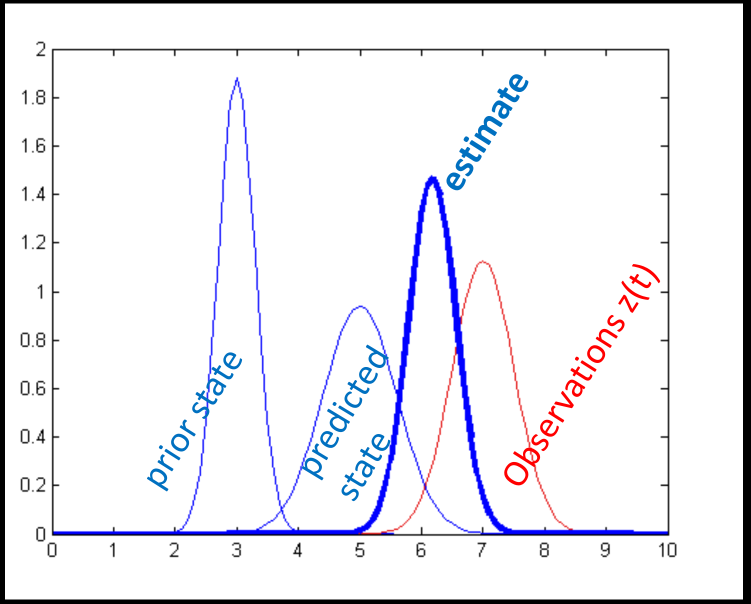


Kalman Filter

State estimate: $\mu(t)$
State uncertainty: $\Sigma(t)$
Process noise: Σ_u
Kalman filter gain: K_{KF}
Measurement noise: Σ_z

- Assume that posterior and prior belief are Gaussian variables
  - Prediction step
    - $x(t) = A\, x(t-1) + Bu(t) + n$, where…
      - $\mu_p(t) = A\, \mu(t-1) + B\, u(t)$
      - $\Sigma_p(t) = A\, \Sigma(t-1)\, A^T + \Sigma_u$
  - Update step
    - $K_{KF} = \Sigma_p(t)\, C^T\, (\, C\, \Sigma_p(t)\, C^T + \Sigma_z)^{-1}$
    - $\mu(t) = \mu_p(t) + K_{KF}\, (\, z(t) - C\, \mu_p(t)\,)$
    - $\Sigma(t) = (\, \mathbf{I} - K_{KF}\, C)\, \Sigma_p(t)$





Kalman Filter

```
Function ( $\mu(t-1), \Sigma(t-1), u(t), z(t)$ )
  1. $\mu_p(t) = A\,\mu(t-1) + B\,u(t)$
  2. $\Sigma_p(t) = A\,\Sigma(t-1)\,A^T + \Sigma_u$
                                        prediction
  3. $K_{KF} = \Sigma_p(t)\,C^T\,(\,C\,\Sigma_p(t)\,C^T + \Sigma_z)^{-1}$
  4. $\mu(t) = \mu_p(t) + K_{KF}\,(\,z(t) - C\,\mu_p(t)\,)$
  5. $\Sigma(t) = (\,I - K_{KF}\,C)\,\Sigma_p(t)$
                                        update
  6. Return $\mu(t)$ and $\Sigma(t)$
```

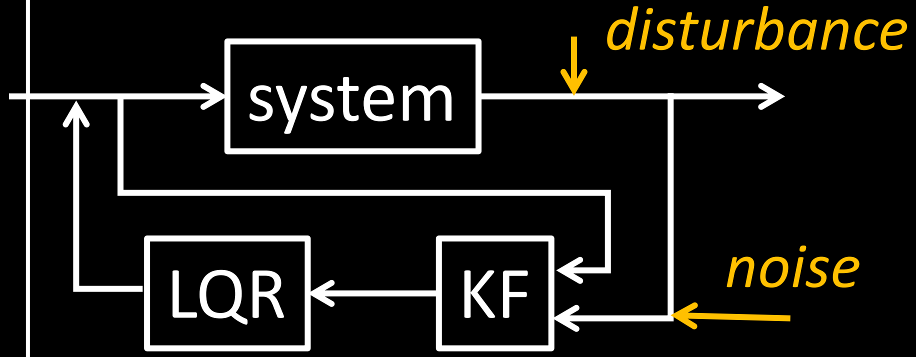
State estimate: $\mu(t)$

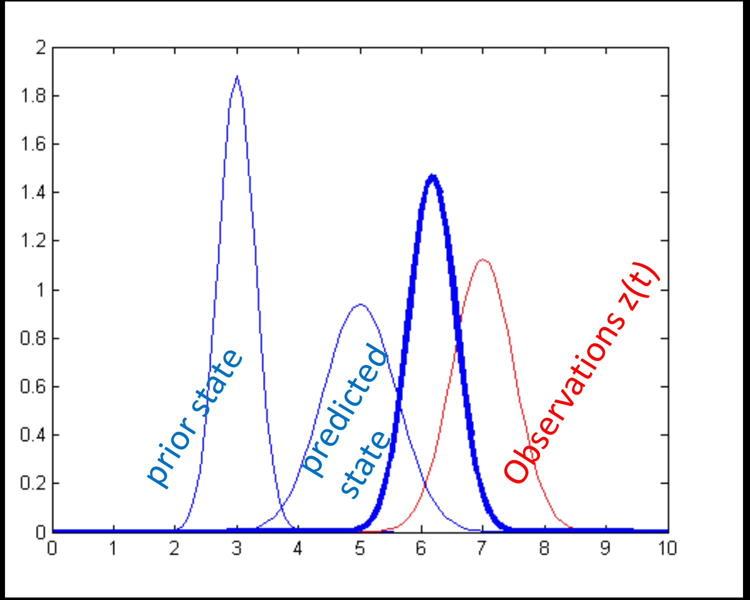
State uncertainty: $\Sigma(t)$

Process noise: Σ_u

Kalman filter gain: K_{KF}

Measurement noise: Σ_z





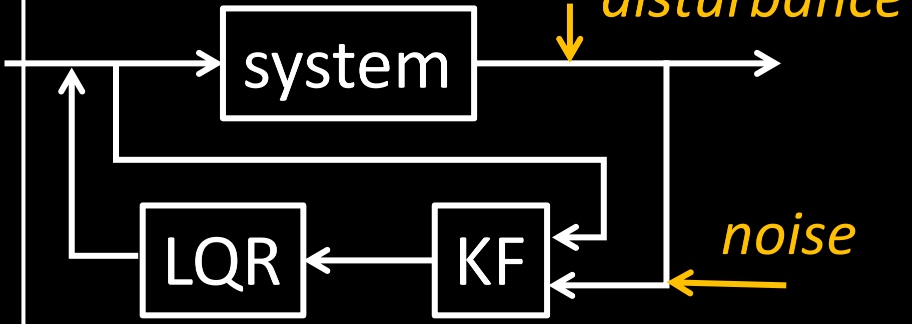
Kalman Filter

```
Kalman Filter ( $\mu(t-1)$, $\Sigma(t-1)$, $u(t)$, $z(t)$ )
1.  $\mu_p(t) = A\,\mu(t-1) + B\,u(t)$
2.  $\Sigma_p(t) = A\,\Sigma(t-1)\,A^T + \Sigma_u$
3.  $K_{KF} = \Sigma_p(t)\,C^T\,(C\,\Sigma_p(t)\,C^T + \Sigma_z)^{-1}$
4.  $\mu(t) = \mu_p(t) + K_{KF}\,(z(t) - C\,\mu_p(t))$
5.  $\Sigma(t) = (I - K_{KF}\,C)\,\Sigma_p(t)$
6.  Return $\mu(t)$ and $\Sigma(t)$
```

prediction (steps 1–2)

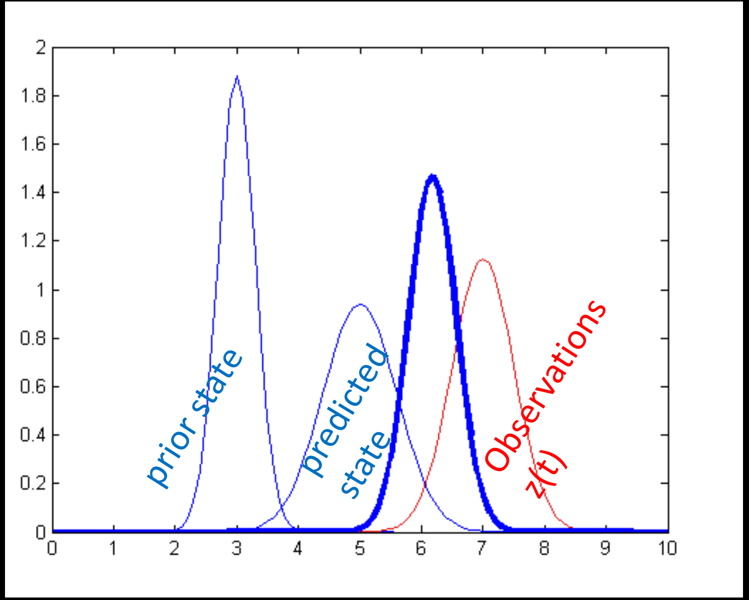
update (steps 3–5)

State estimate: $\mu(t)$
State uncertainty: $\Sigma(t)$
Process noise: Σ_u
Kalman filter gain: K_{KF}
Measurement noise: Σ_z



Example process and measurement noise covariance matrices:

$$\Sigma_u = \begin{bmatrix} {\sigma_1}^2 & 0 \\ 0 & {\sigma_2}^2 \end{bmatrix}, \Sigma_z = {\sigma_3}^2$$



Kalman Filter

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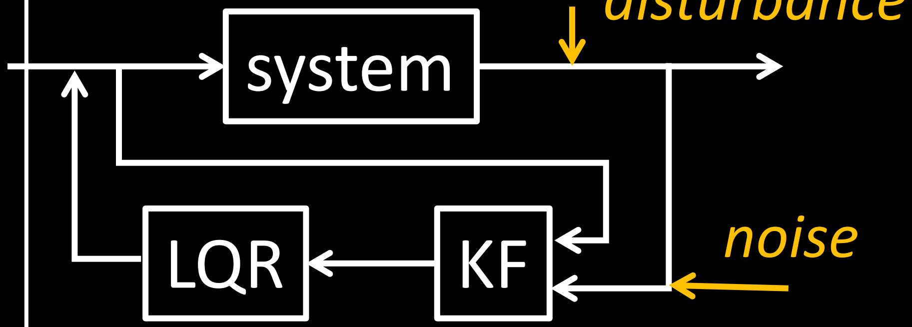
Kalman Filter ( $\mu(t-1), \Sigma(t-1), u(t), z(t)$ )
1.  $\mu_p(t) = A\,\mu(t-1) + B\,u(t)$
2.  $\Sigma_p(t) = A\,\Sigma(t-1)\,A^T + \Sigma_u$
3.  $K_{KF} = \Sigma_p(t)\,C^T\,(\,C\,\Sigma_p(t)\,C^T + \Sigma_z)^{-1}$
4.  $\mu(t) = \mu_p(t) + K_{KF}\,(\,z(t) - C\,\mu_p(t)\,)$
5.  $\Sigma(t) = (\,\mathbf{I} - K_{KF}\,C)\,\Sigma_p(t)$
6.  Return $\mu(t)$ and $\Sigma(t)$
```

prediction

update

State estimate: $\mu(t)$
State uncertainty: $\Sigma(t)$
Process noise: Σ_u
Kalman filter gain: K_{KF}
Measurement noise: Σ_z



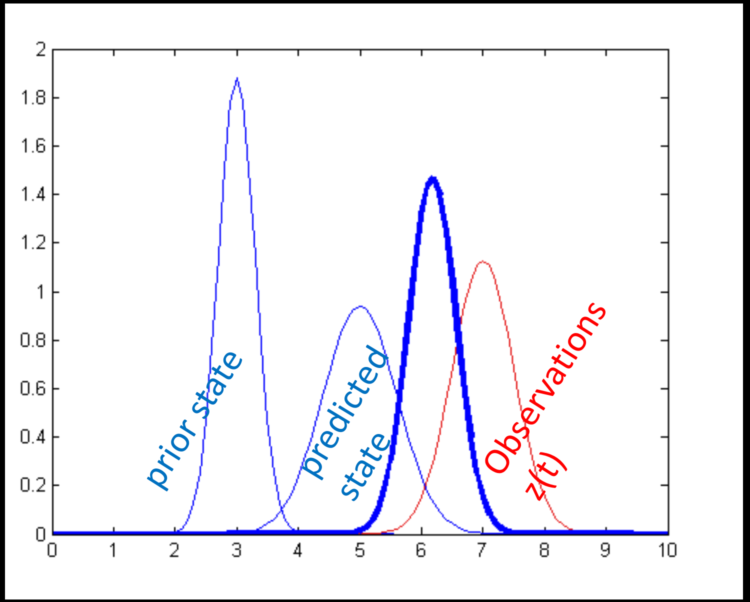
disturbance

system

LQR

KF

noise



Example process and measurement noise covariance matrices:

$$\Sigma_u = \begin{bmatrix} {\sigma_1}^2 & 0 \\ 0 & {\sigma_2}^2 \end{bmatrix}, \Sigma_z = {\sigma_3}^2$$

Lab 7: Kalman Filter

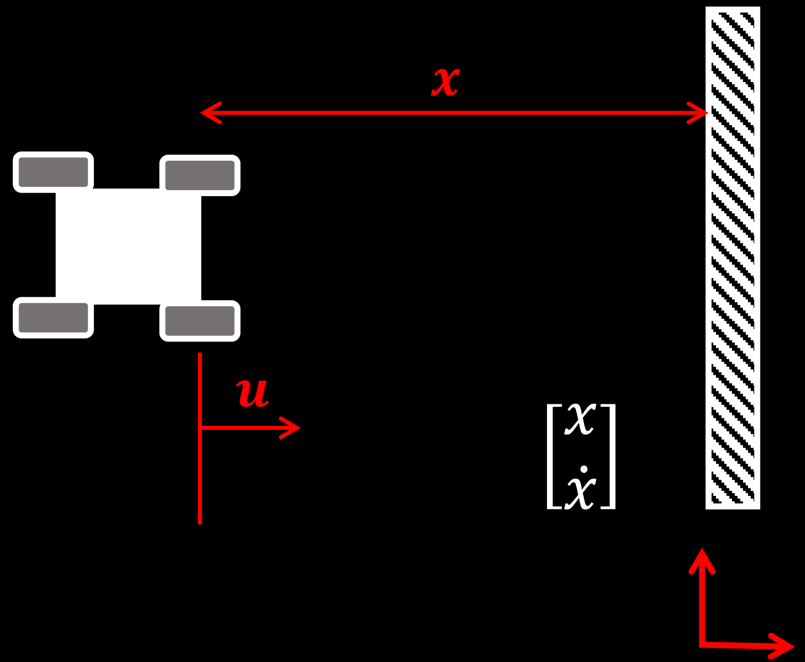
$$F = ma = m\ddot{x}$$

$$F = u - d\dot{x}$$

$$u - d\dot{x} = m\ddot{x}$$

$$\ddot{x} = \frac{u}{m} - \frac{d}{m}\dot{x}$$

What is d and m?



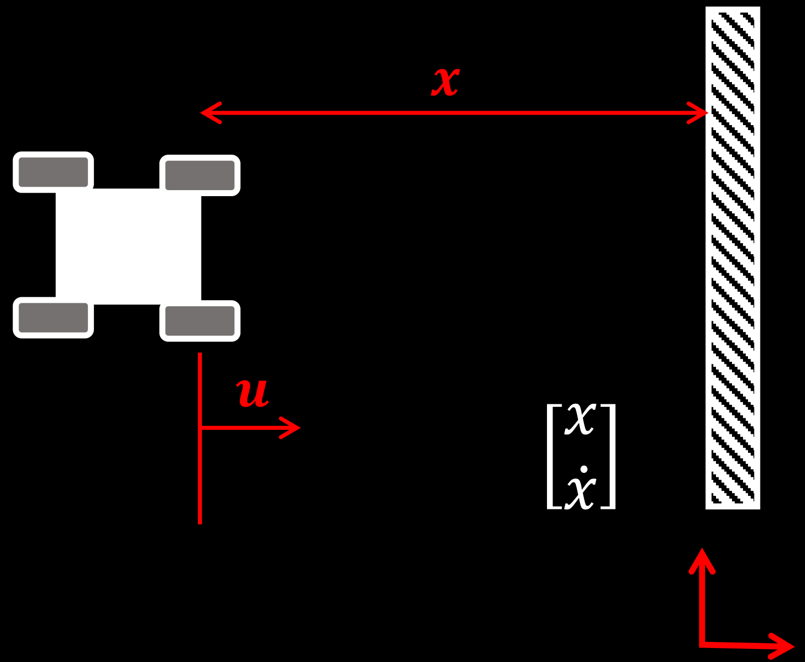
State space equation

$$\begin{bmatrix}\dot{x}\\\ddot{x}\end{bmatrix} = \begin{bmatrix}0 & 1\\0 & -\frac{d}{m}\end{bmatrix}\begin{bmatrix}x\\\dot{x}\end{bmatrix} + \begin{bmatrix}0\\\frac{1}{m}\end{bmatrix}u$$

$$C = \begin{bmatrix}-1 & 0\end{bmatrix}$$

Lab 7: Kalman Filter

$$F = ma = m\ddot{x}$$
$$F = u - d\dot{x}$$
$$u - d\dot{x} = m\ddot{x}$$
$$\ddot{x} = \frac{u}{m} - \frac{d}{m}\dot{x}$$



What is d and m?

- At steady state (cst speed), we can find *d*
  - $0 = \frac{u_{ss}}{m} - \frac{d}{m}\dot{x}_{ss}$
  - $0 = \frac{u_{ss}}{m} - \frac{d}{m}\dot{x}_{ss} \quad \leftrightarrow \quad d = \frac{u_{ss}}{\dot{x}_{ss}}$

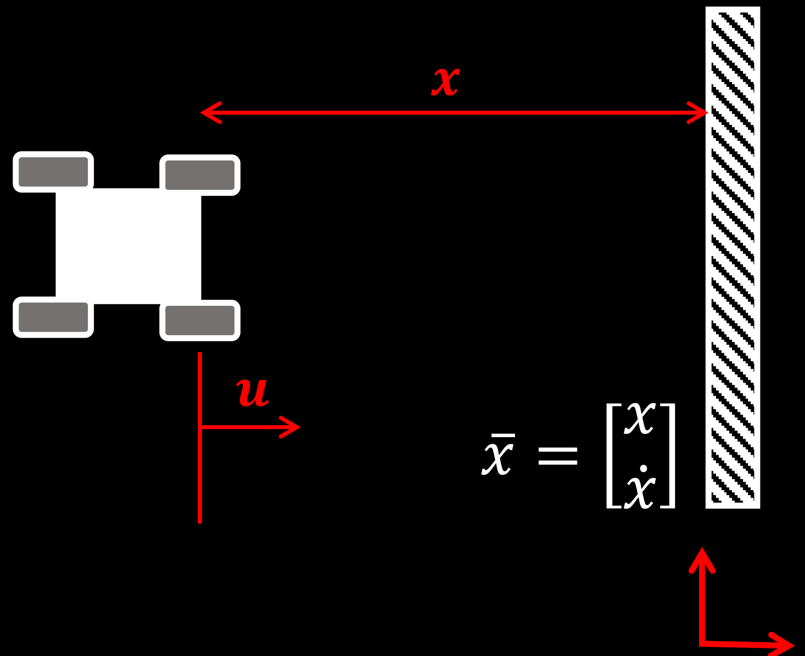
State space equation

$$\begin{bmatrix}\dot{x}\\ \ddot{x}\end{bmatrix} = \begin{bmatrix}0 & 1\\ 0 & -\frac{d}{m}\end{bmatrix}\begin{bmatrix}x\\ \dot{x}\end{bmatrix} + \begin{bmatrix}0\\ \frac{1}{m}\end{bmatrix}u$$

$$C = \begin{bmatrix}-1 & 0\end{bmatrix}$$

Lab 7: Kalman Filter

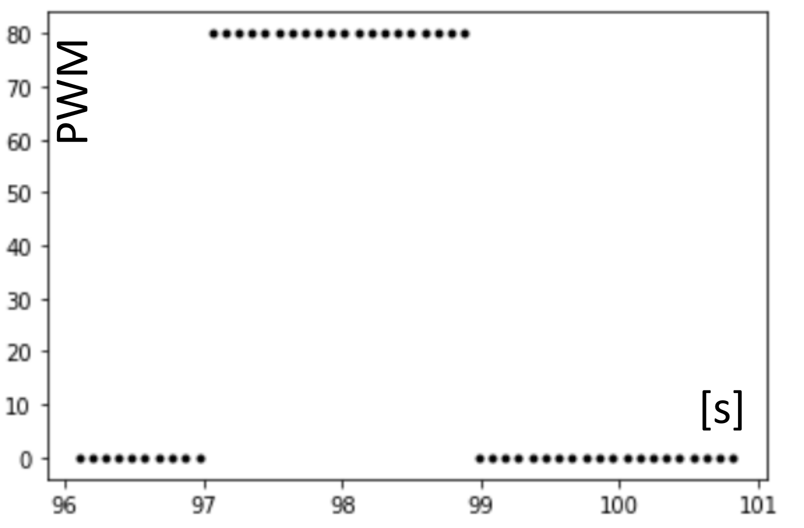
$$F = ma = m\ddot{x}$$
$$F = u - d\dot{x}$$
$$u - d\dot{x} = m\ddot{x}$$
$$\ddot{x} = \frac{u}{m} - \frac{d}{m}\dot{x}$$

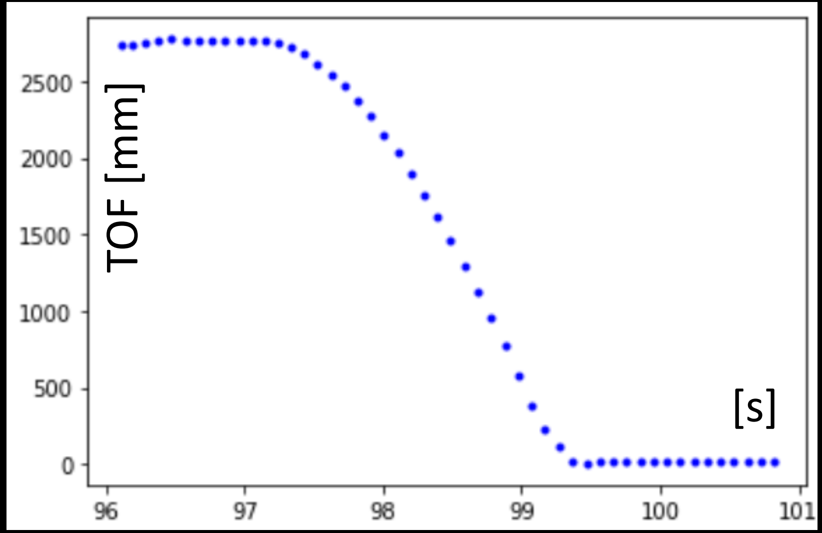


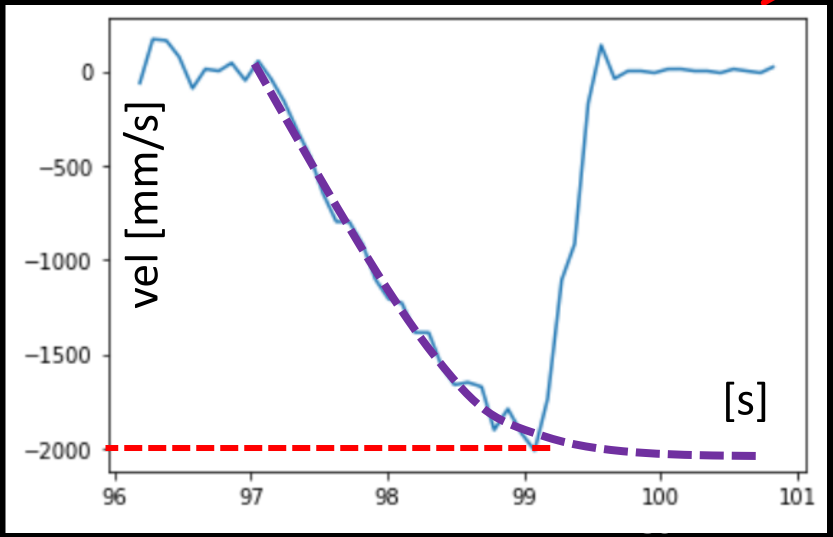
$$\bar{x} = \begin{bmatrix} x \\ \dot{x} \end{bmatrix}$$

What is d and m?

- At steady state (cst speed), we can find *d*







Lab 7: Kalman Filter

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$$F = u - d\dot{x}$$
$$u - d\dot{x} = m\ddot{x}$$
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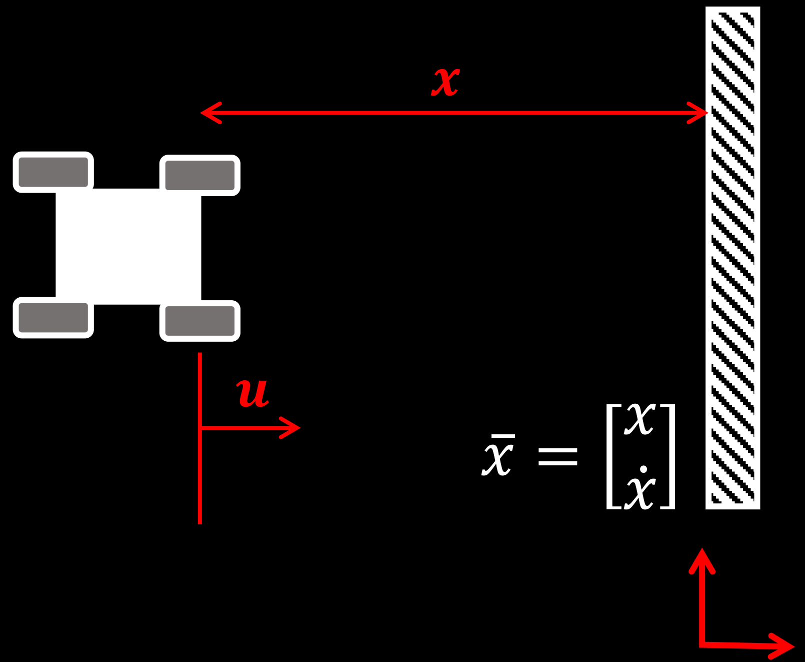
What is d and m?

- At steady state (cst speed), we can find *d*
  - $0 = \frac{u_{ss}}{m} - \frac{d}{m}\dot{x}$
  - $0 = \frac{u_{ss}}{m} - \frac{d}{m}\dot{x} \quad \leftrightarrow \quad d = \frac{u_{ss}}{\dot{x}}$
  - $d \approx \frac{u_{ss}}{2000mm/s}$ *(Assume u=1 for now)*



$$\bar{x} = \begin{bmatrix} x \\ \dot{x} \end{bmatrix}$$

State space equation

$$\begin{bmatrix} \dot{x} \\ \ddot{x} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & -\frac{d}{m} \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix} u$$

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Lab 7: Kalman Filter

$$F = ma = m\ddot{x}$$

$$F = u - d\dot{x}$$

$$u - d\dot{x} = m\ddot{x}$$

$$\ddot{x} = \frac{u}{m} - \frac{d}{m}\dot{x}$$

1st order system:

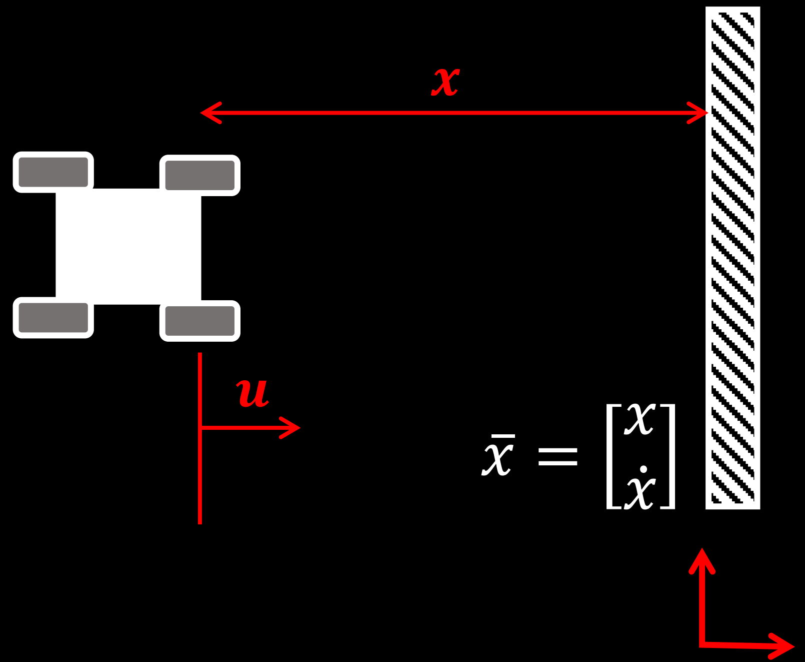
$$\frac{dy(t)}{dt} + ky(t) = ru(t)$$

Unit step response solution:

$$y(t) = \frac{r}{k}(1 - e^{-kt})$$

What is d and m?

- Use the rise time to determine m
  - $\dot{v}(t) + \frac{d}{m}v(t) = \frac{1}{m}u(t)$
  - $v(t) = \frac{1}{d}(1 - e^{-\frac{d}{m}t}) \leftrightarrow 1 - dv(t) = e^{-\frac{d}{m}t_{0.9}}$
  - $\ln(1 - dv(t)) = -\frac{d}{m}t$
  - $m = \frac{-dt}{\ln(1-dv(t))}$



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Lab 7: Kalman Filter

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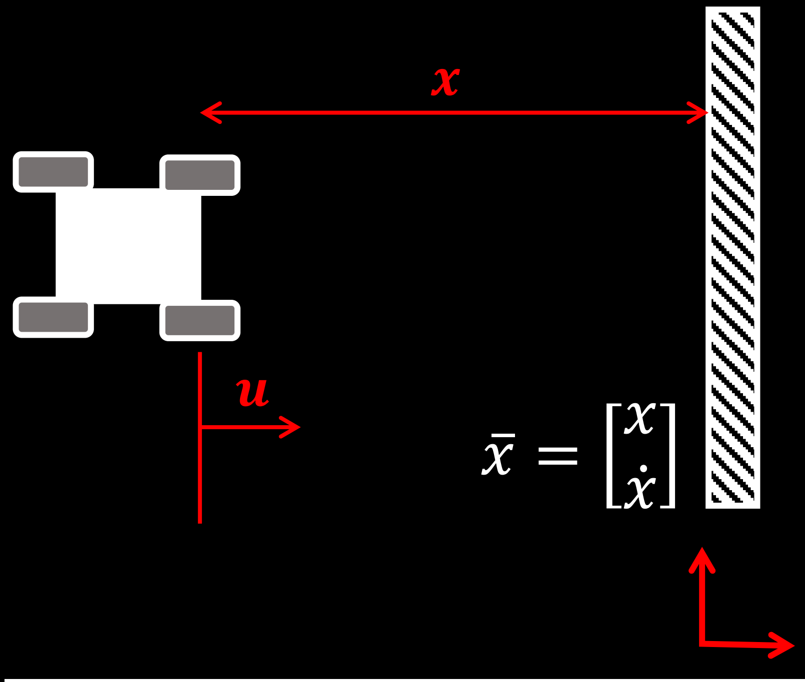
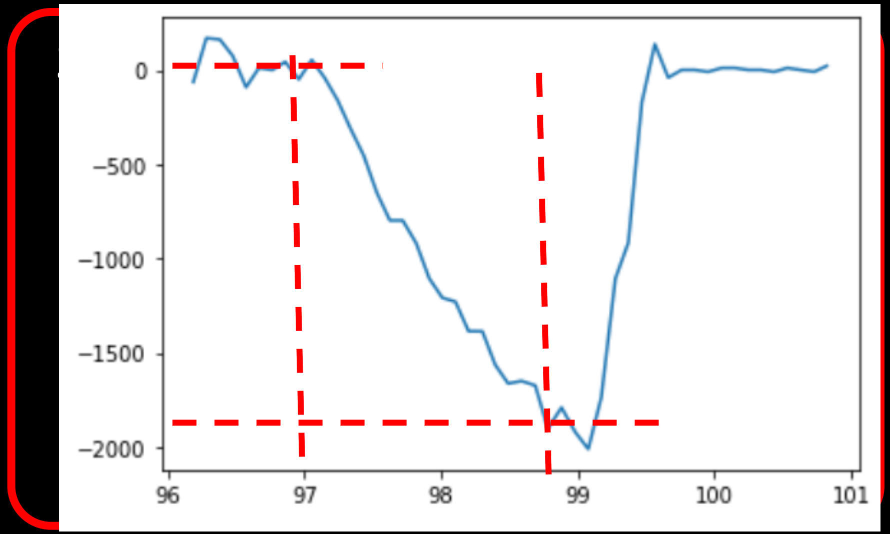


$$\bar{x} = \begin{bmatrix} x \\ \dot{x} \end{bmatrix}$$

What is d and m?

- Use the rise time to determine m
  - $\dot{v}(t) + \frac{d}{m}v(t) = \frac{1}{m}u(t)$
  - $m = \frac{-dt}{\ln(1-dv(t))}$
  - $m = \frac{-0.0005\cdot 1.9}{\ln(0.1)} = 4.1258 \cdot 10^{-4}$

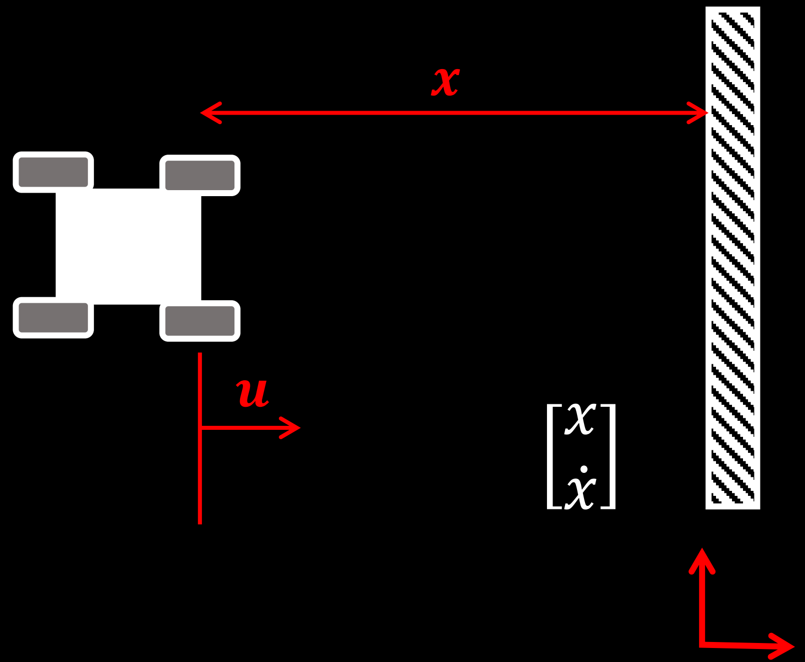


Lab 7: Kalman Filter

$$F = ma = m\ddot{x}$$
$$F = u - d\dot{x}$$
$$u - d\dot{x} = m\ddot{x}$$
$$\ddot{x} = \frac{u}{m} - \frac{d}{m}\dot{x}$$

What is d and m?

- At steady state (cst speed), we can find *d*
  - $d = \frac{u}{\dot{x}} \approx 0.0005$ *(Assume u=1 for now)*
- We can use the rise time to find *m*
  - $m = \frac{-dt_{0.9}}{\ln(0.1)} \approx 4.1258 \cdot 10^{-4}$



State space equation

$$\begin{bmatrix} \dot{x} \\ \ddot{x} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & -\frac{d}{m} \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix} u$$

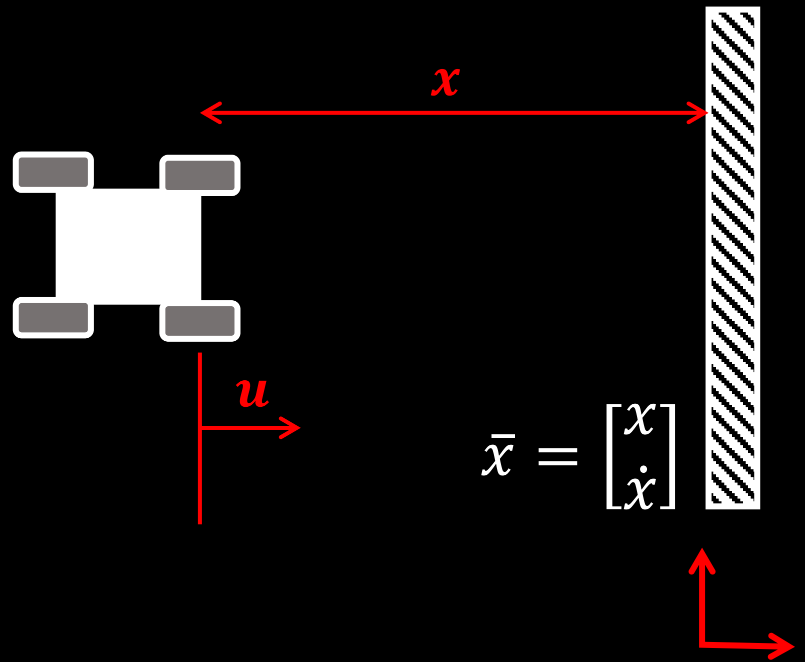
$$C = \begin{bmatrix} -1 & 0 \end{bmatrix}$$

Lab 7: Kalman Filter

- We have A, B, C, Σ_u, Σ_z

- Discretize the A and B matrices
  - x(n+1) = x(n) + dx
  - dx/dt = Ax+Bu ⇔ dx = dt (Ax + Bu)
  - x(n+1) = x(n) + dt (Ax(n) + Bu)
  - $x(n+1) = \underbrace{(I + dt*A)}_{A_d}\, x(n) + \underbrace{dt*B}_{B_d}\, u$
  - dt is our sampling time (0.130s)

- Rescale from unity input to actual input



$$\bar{x} = \begin{bmatrix} x \\ \dot{x} \end{bmatrix}$$

State space equation

$$\begin{bmatrix} \dot{x} \\ \ddot{x} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & -\frac{d}{m} \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix} u$$

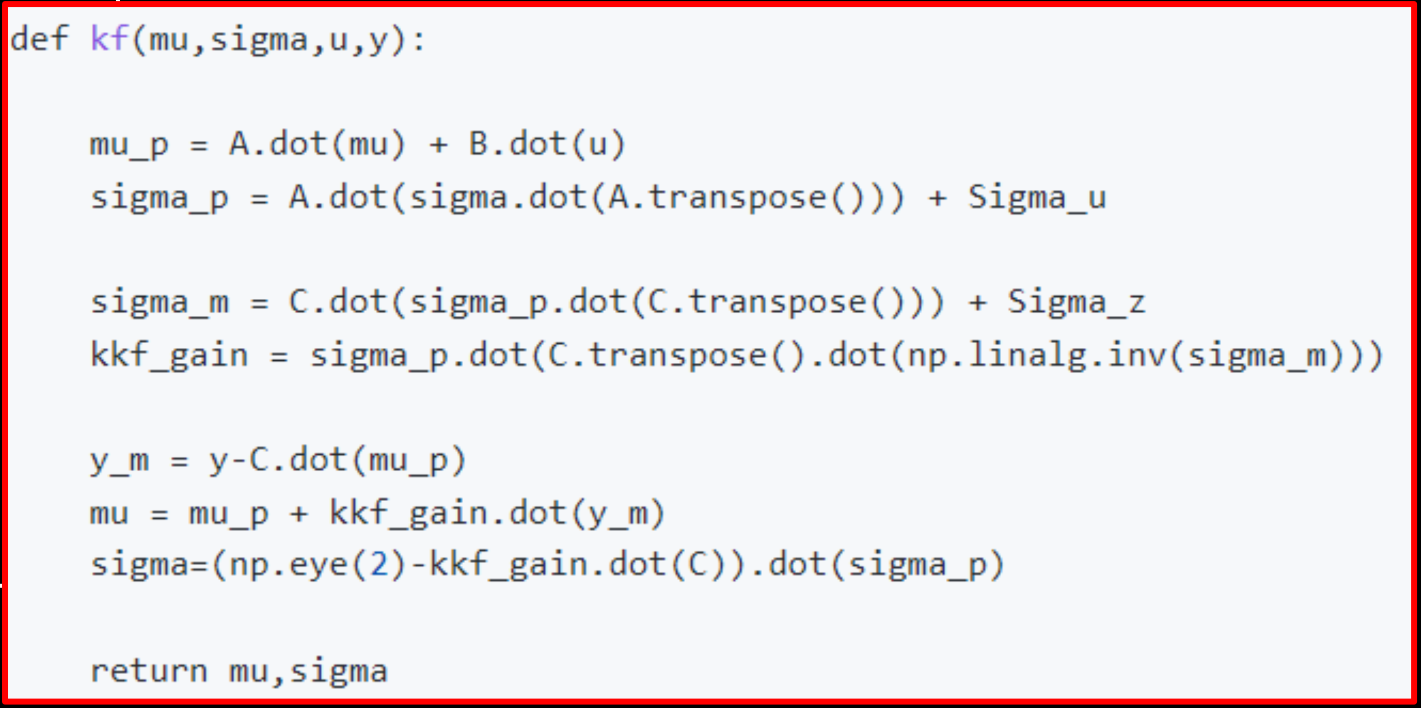
$$C = \begin{bmatrix} -1 & 0 \end{bmatrix}$$

Lab 7: Kalman Filter

Implement the Kalman Filter

Next, determine measurement and process noise

```
Kalman Filter ( $\mu$(t-1), $\Sigma$(t-1), u(t), z(t) )
1.  $\mu_p(t) = A\,\mu(t-1) + B\,u(t)$
2.  $\Sigma_p(t) = A\,\Sigma(t-1)\,A^T + \Sigma_u$
3.  $K_{KF} = \Sigma_p(t)\,C^T\,(\,C\,\Sigma_p(t)\,C^T + \Sigma_z)^{-1}$
4.  $\mu(t) = \mu_p(t) + K_{KF}\,(\,z(t) - C\,\mu_p(t)\,)$
5.  $\Sigma(t) = (\,\mathbf{I} - K_{KF}\,C)\,\Sigma_p(t)$
6.  Return $\mu(t)$ and $\Sigma(t)$
```



```
def kf(mu,sigma,u,y):

    mu_p = A.dot(mu) + B.dot(u)
    sigma_p = A.dot(sigma.dot(A.transpose())) + Sigma_u

    sigma_m = C.dot(sigma_p.dot(C.transpose())) + Sigma_z
    kkf_gain = sigma_p.dot(C.transpose().dot(np.linalg.inv(sigma_m)))

    y_m = y-C.dot(mu_p)
    mu = mu_p + kkf_gain.dot(y_m)
    sigma=(np.eye(2)-kkf_gain.dot(C)).dot(sigma_p)

    return mu,sigma
```

Lab 7: Kalman Filter

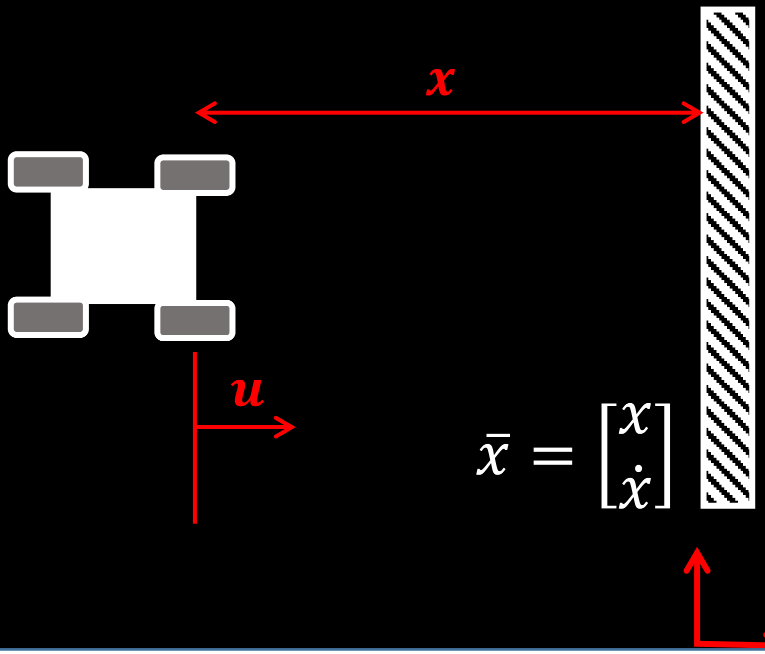
Implement the Kalman Filter

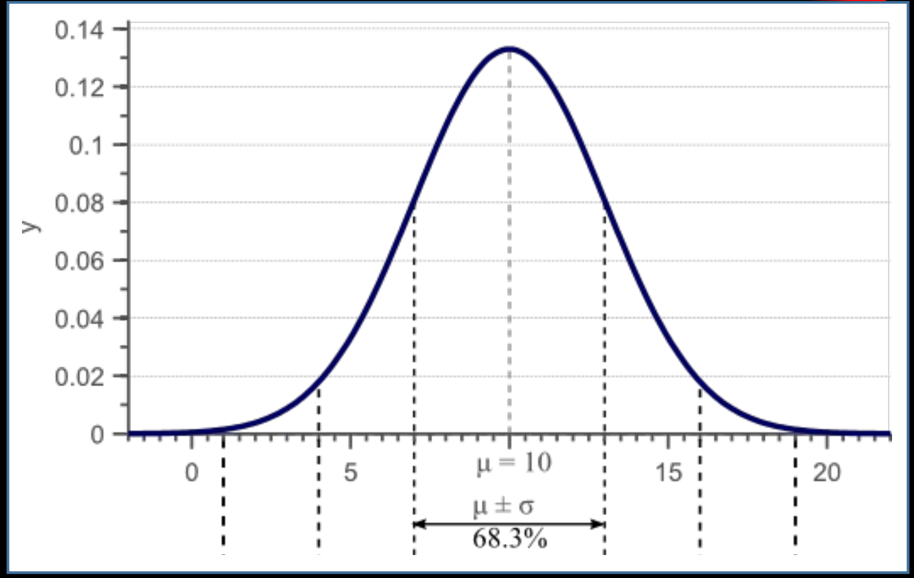
- Measurement noise
    - $\Sigma_z = [{\sigma_3}^2]$
    - $\sigma_3^2 = (20mm)^2$
- Process noise (dependent on sampling rate)

$$\Sigma_u = \begin{bmatrix} {\sigma_1}^2 & 0 \\ 0 & {\sigma_2}^2 \end{bmatrix}$$

- Trust in modeled position:
    - $\text{Pos}_{\text{stddev}}$ after 1s: $\sqrt{10^2 \cdot \frac{1}{0.13}} = 27.7mm$
- Trust in modeled speed:
    - $\text{Speed}_{\text{stddev}}$ after 1s: $\sqrt{10^2 \cdot \frac{1}{0.13}} = 27.7mm/s$

1 sample per ~0.13s





Lab 7: Kalman Filter

Implement the Kalman Filter

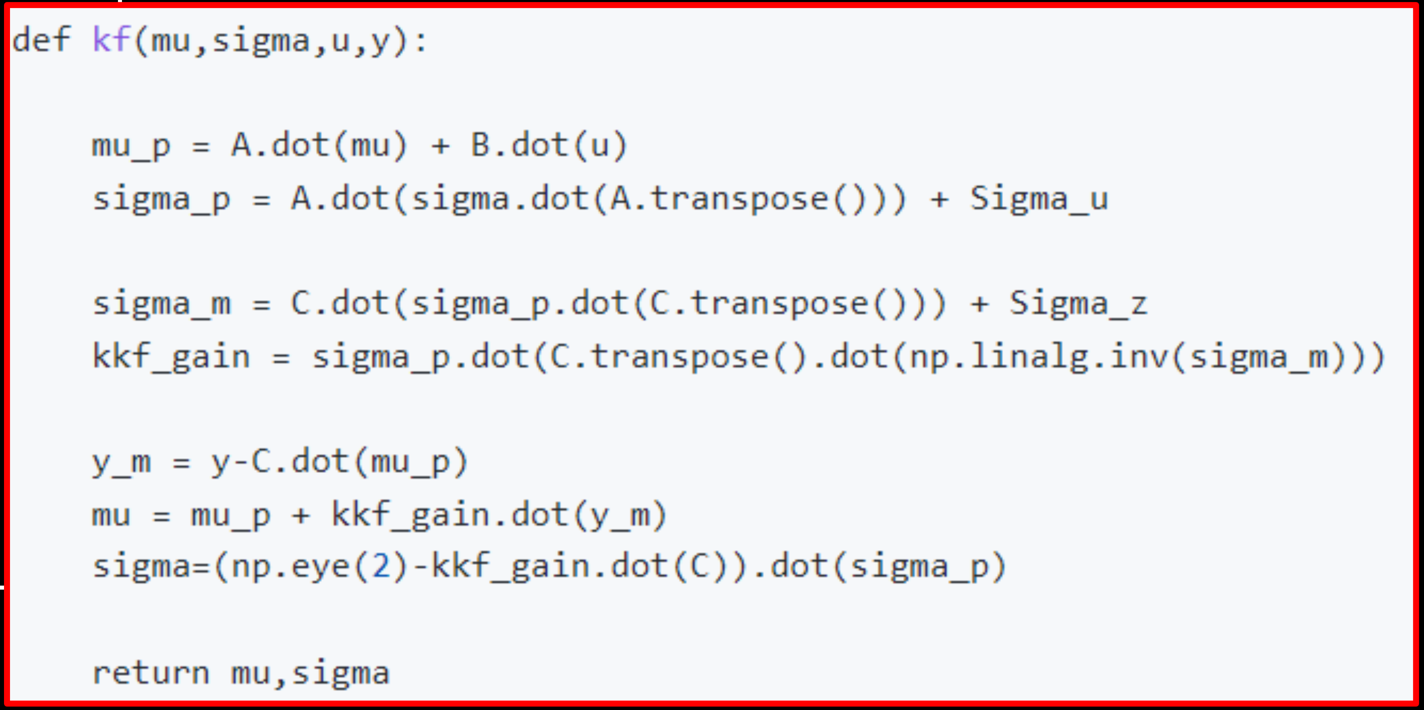
```
Kalman Filter ( $\mu(t-1), \Sigma(t-1), u(t), z(t)$ )
1. $\mu_p(t) = A\,\mu(t-1) + B\,u(t)$
2. $\Sigma_p(t) = A\,\Sigma(t-1)\,A^T + \Sigma_u$
3. $K_{KF} = \Sigma_p(t)\,C^T\,(\,C\,\Sigma_p(t)\,C^T + \Sigma_z)^{-1}$
4. $\mu(t) = \mu_p(t) + K_{KF}\,(\,z(t) - C\,\mu_p(t)\,)$
5. $\Sigma(t) = (\,\mathbf{I} - K_{KF}\,C)\,\Sigma_p(t)$
6. Return $\mu(t)$ and $\Sigma(t)$
```

Finally, determine your initial state mean and covariance

$\mu(t-1)$

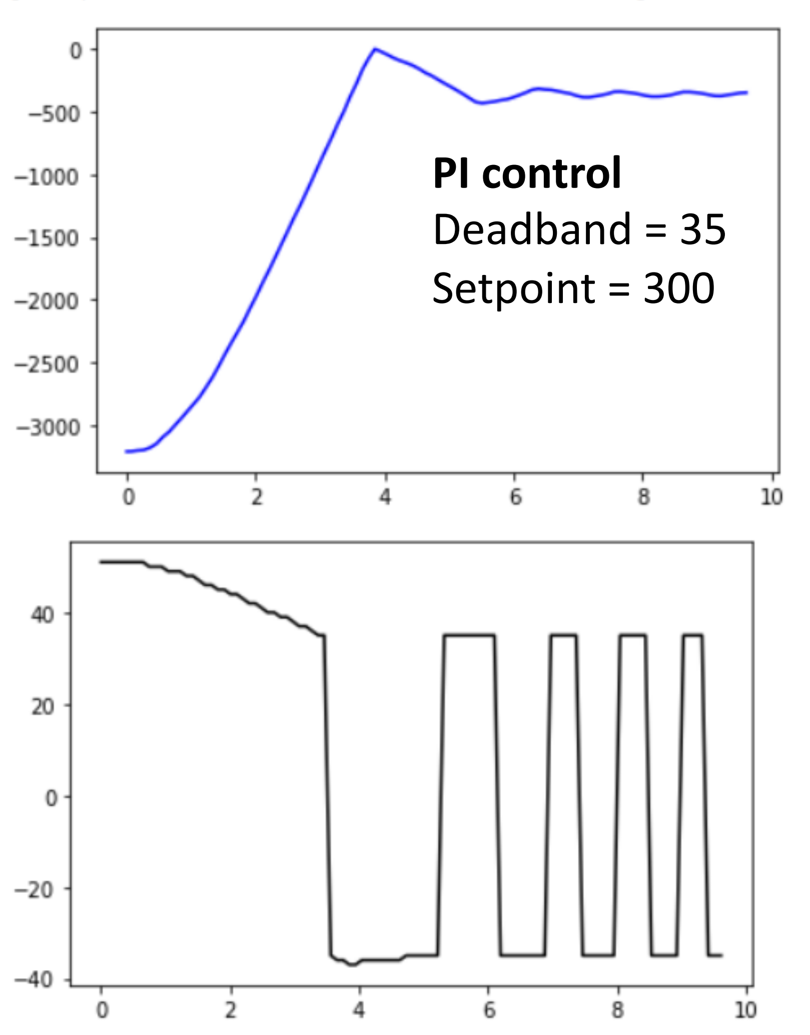
$\Sigma(t-1)$

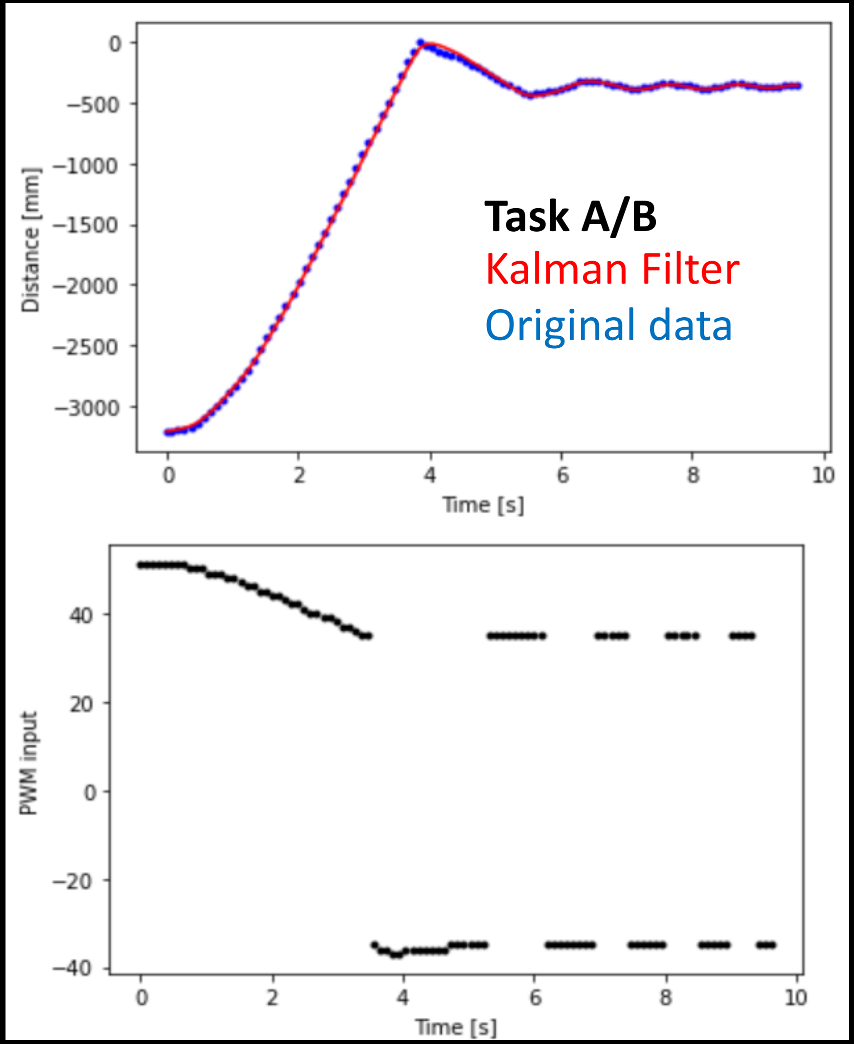
Play video!!



```python
def kf(mu,sigma,u,y):

    mu_p = A.dot(mu) + B.dot(u)
    sigma_p = A.dot(sigma.dot(A.transpose())) + Sigma_u

    sigma_m = C.dot(sigma_p.dot(C.transpose())) + Sigma_z
    kkf_gain = sigma_p.dot(C.transpose().dot(np.linalg.inv(sigma_m)))

    y_m = y-C.dot(mu_p)
    mu = mu_p + kkf_gain.dot(y_m)
    sigma=(np.eye(2)-kkf_gain.dot(C)).dot(sigma_p)

    return mu,sigma
```

Lab 7: Kalman Filter


PI control
Deadband = 35
Setpoint = 300


Task A/B
Kalman Filter
Original data