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ECE 4160/5160
MAE 4910/5910

Fast Robots

Motion models

FastRobots-2023

ECE4160/5160-MAE 4190/5190: Fast Robots course, offered at Cornell University in Spring 2023

View On GitHub

This project is maintained by CEI-lab

Hosted on GitHub Pages using the Dinky theme

Fast Robots @Cornell, Spring 2023

Return to main page

Lab 8 Stunts!

Objective

The purpose of this lab is to combine everything you've done up till now to do fast stunts. *This* is the reason you labored all those long hours in the lab carefully soldering up and mounting your components! Your grade will be based partially on your hardware/software design and partially on how fast your robot manages to complete the stunt (relative to everyone else in class). We will also have everyone vote on the coolest stunt and the best blooper video - the top picks will receive up to 2 bonus points.

Parts Required

- 1 x R/C stunt car

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Fast Robots

Bayes Filter and Motion Models

Markov Assumption

The Markov assumption postulates that past and future data are independent if one knows the current state

- State generative model
  - $p(x_t \mid x_{0:t-1}, z_{1:t-1}, u_{1:t}) = p(x_t \mid x_{t-1}, u_t)$
- Measurement generative model
  - $p(z_t \mid x_{0:t}, z_{1:t-1}, u_{1:t}) = p(z_t \mid x_t)$

Andrey Markov (1856–1922) was a Russian mathematician best known for his work on stochastic processes

Robot-Environment Model

+

Markov Assumption

+

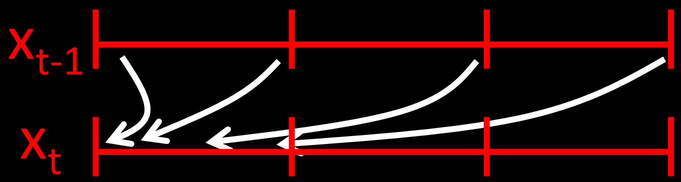
Bayes Theorem

=

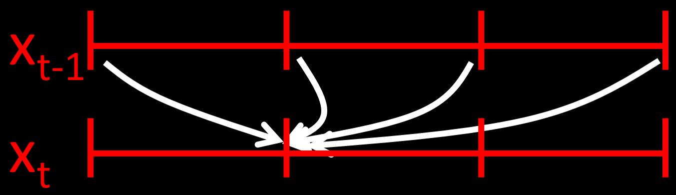
Bayes Filter

Bayes Filter

First for-loop iteration



Second for-loop iteration



....

Correct for likelihood of sensor measurement

1. **Algorithm Bayes_Filter** $(bel(x_{t-1}), u_t, z_t)$:
2. for all x_t do
3. $\overline{bel}(x_t) = \sum_{x_{t-1}} p(x_t | u_t, x_{t-1})\ bel(x_{t-1})$ (Prediction step)
4. $bel(x_t) = \eta\, p(z_t | x_t)\ \overline{bel}(x_t)$ (Update step)
5. endfor
6. return $bel(x_t)$

Transition probability /action model

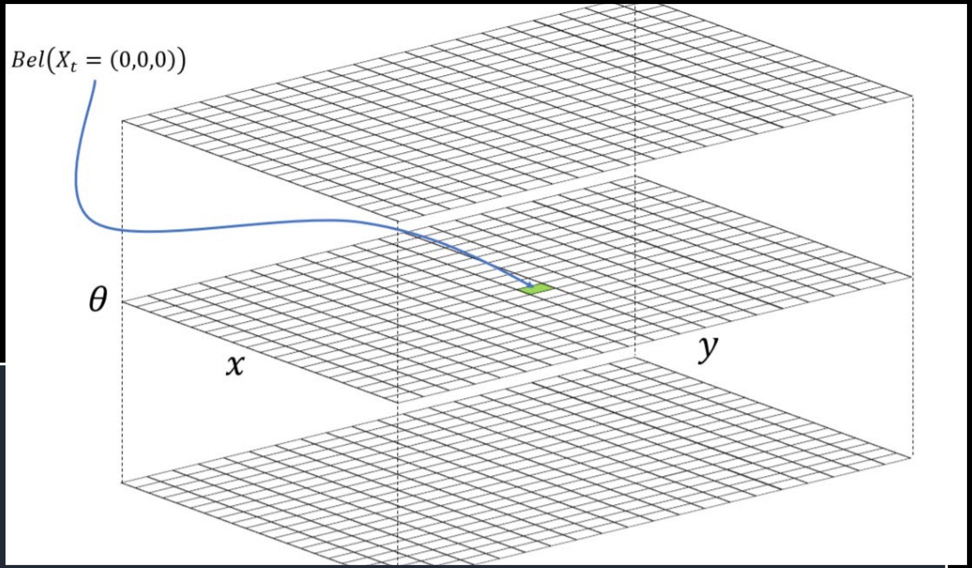
prior

Measurement Probability / Sensor Model

Normalization constant

Bayes Filter

This is a lot of computation!



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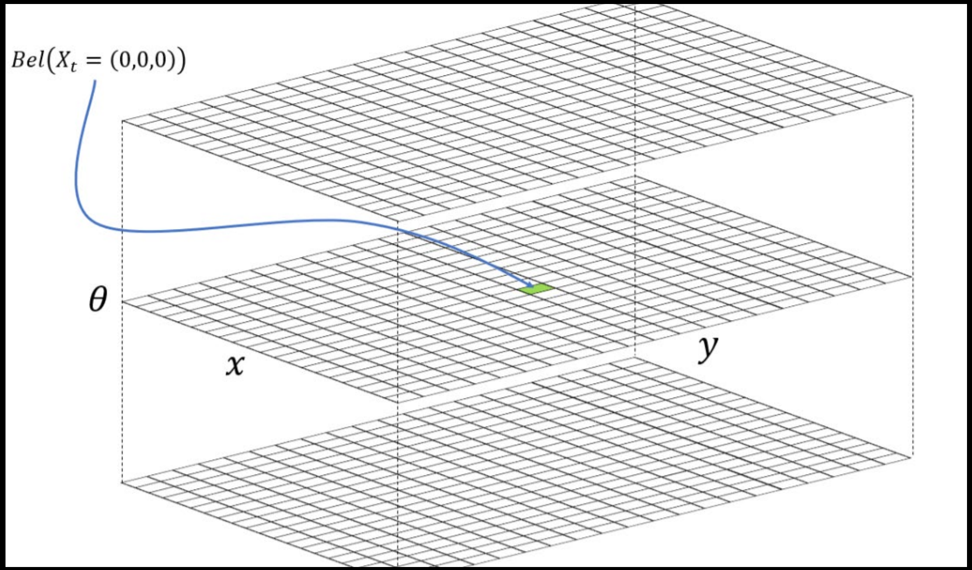
Violations of Markov Assumption



This is a lot of computation!

- Typical violations of the Markov assumption
  - Environmental dynamics not included in x_t
  - Inaccuracies in the probabilistic models $p(z_t \mid x_t)$ and $p(x_t \mid u_t, x_{t-1})$
  - Approximation errors when representing belief functions
- Incomplete state representations are often preferable to reduce computational complexity of the Bayes filter algorithm
- *In practice Bayes filters have been found to be surprisingly robust to such violations*

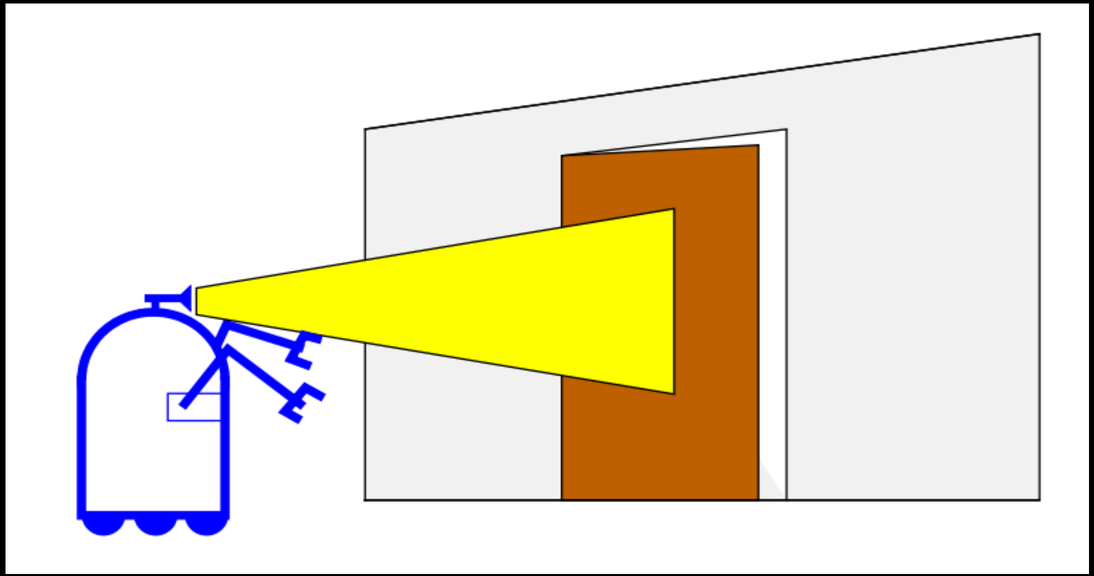
Bayes Filter

Example 1

Bayes Filter - Example 1

- A robot can "*observe*" a door through its sensor and can interact with it by "*pushing*"
- The door may be in one of two states
  - *open* or *closed*
- At any given time, the robot can either
  - *push* or *do_nothing*
- The sensors and the actuators on the robot are noisy

- The probability that the robot can sense an *open* door is 0.6
- The probability that the robot can sense a *closed* door is 0.8
- After a *push* action, probability that a door is *open* if it was previously open is 1
- After a *push* action, probability that a door is *open* if it was previously closed is 0.8
- If the robot *does nothing*, the door continues to be in the previous state



Bayes Filter - Example 1

- Measurement model
  - $p(Z_t = closed \mid X_t = is\_closed) = 0.8$
  - $p(Z_t = open \mid X_t = is\_closed) = 0.2$
  - $p(Z_t = closed \mid X_t = is\_open) = 0.4$
  - $p(Z_t = open \mid X_t = is\_open) = 0.6$

- The probability that the robot can sense an *open* door is 0.6
- The probability that the robot can sense a *closed* door is 0.8
- After a *push* action, probability that a door is *open* if it was previously open is 1
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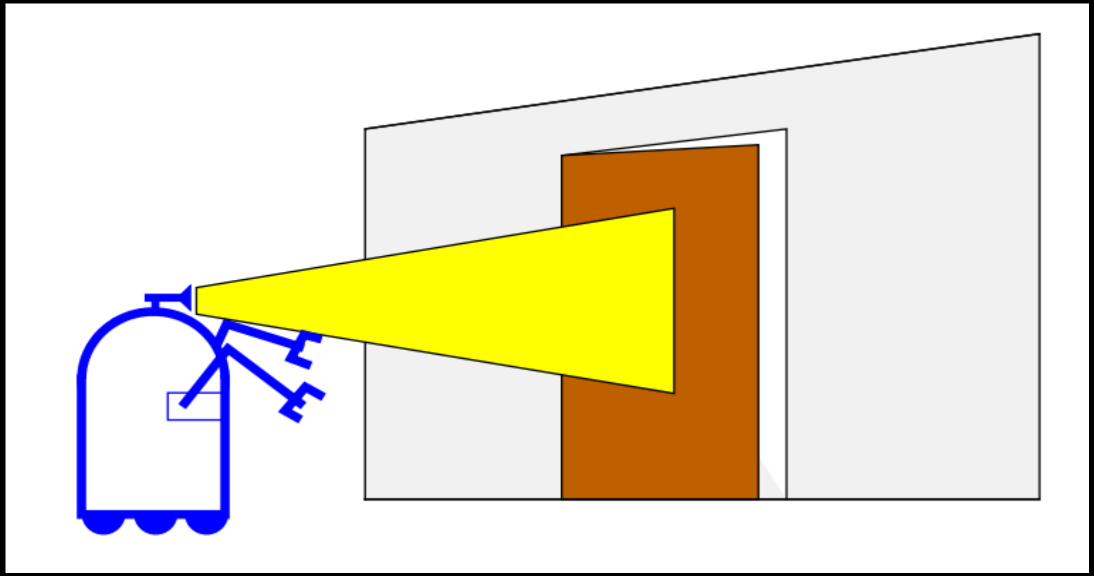

Bayes Filter - Example 1

Action model

- $p(X_t = is\_closed | U_t = do\_nothing, X_{t-1} = is\_closed)$ =1
- $p(X_t = is\_open | U_t = do\_nothing, X_{t-1} = is\_closed)$ =0
- $p(X_t = is\_closed | U_t = do\_nothing, X_{t-1} = is\_open)$ =0
- $p(X_t = is\_open | U_t = do\_nothing, X_{t-1} = is\_open)$ =1

- $p(X_t = is\_closed | U_t = push, X_{t-1} = is\_closed)$ =0.2
- $p(X_t = is\_open | U_t = push, X_{t-1} = is\_closed)$ =0.8
- $p(X_t = is\_closed | U_t = push, X_{t-1} = is\_open)$ =0
- $p(X_t = is\_open | U_t = push, X_{t-1} = is\_open)$ =1

- The probability that the robot can sense an *open* door is 0.6
- The probability that the robot can sense a *closed* door is 0.8
- After a *push* action, probability that a door is *open* if it was previously open is 1
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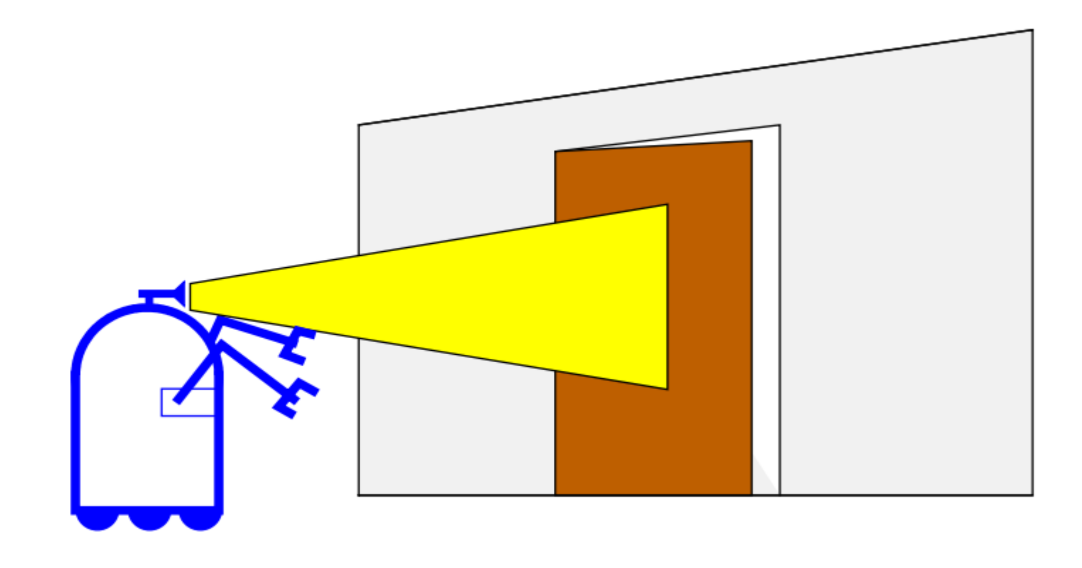

Bayes Filter - Example 1

Initial Conditions

$$bel(X_0 = closed) = bel(X_0 = open) = 0.5$$

Measurement Probability

$$p(Z_t = closed | X_t = is\_closed) = 0.8$$
$$p(Z_t = open | X_t = is\_closed) = 0.2$$
$$p(Z_t = closed | X_t = is\_open) = 0.4$$
$$p(Z_t = open | X_t = is\_open) = 0.6$$

Control Action/Transition Probability

$$p(X_t = is\_closed | U_t = do\_nothing, X_{t-1} = is\_closed) = 1$$
$$p(X_t = is\_open | U_t = do\_nothing, X_{t-1} = is\_closed) = 0$$
$$p(X_t = is\_closed | U_t = do\_nothing, X_{t-1} = is\_open) = 0$$
$$p(X_t = is\_open | U_t = do\_nothing, X_{t-1} = is\_open) = 1$$

$$p(X_t = is\_closed | U_t = push, X_{t-1} = is\_closed) = 0.2$$
$$p(X_t = is\_open | U_t = push, X_{t-1} = is\_closed) = 0.8$$
$$p(X_t = is\_closed | U_t = push, X_{t-1} = is\_open) = 0$$
$$p(X_t = is\_open | U_t = push, X_{t-1} = is\_open) = 1$$

Bayes Filter - Example 1

$u_1 = do\_nothing \text{ and } z_1 = sense\_open$

Incorporate the action

$$\overline{bel}(x_1) = \sum_{x_0} p(x_1|u_1, x_0)\, bel(x_0)$$

Bayes Filter - Example 1

$u_1 = do\_nothing \text{ and } z_1 = sense\_open$

Incorporate the action

$$\overline{bel}(x_1) = \sum_{x_0} p(x_1|u_1, x_0)\, bel(x_0)$$

$$= p(x_1|U_1 = do\_nothing, X_0 = is\_open)\, bel(X_0 = is\_open)$$

$$+ p(x_1|U_1 = do\_nothing, X_0 = is\_closed)\, bel(X_0 = is\_closed)$$

Bayes Filter - Example 1

$u_1 = do\_nothing \; and \; z_1 = sense\_open$

Incorporate the action

$$\overline{bel}(x_1) = \sum_{x_0} p(x_1|u_1, x_0)\, bel(x_0)$$

$$= p(x_1|U_1 = do\_nothing, X_0 = is\_open)\, bel(X_0 = is\_open)$$

$$+ \; p(x_1|U_1 = do\_nothing, X_0 = is\_closed)\, bel(X_0 = is\_closed)$$

For the hypothesis $X_1 = is\_closed$:

$$\overline{bel}(X_1 = is\_closed) = p(X_1 = is\_closed|U_1 = do\_nothing, X_0 = is\_open)\, bel(X_0 = is\_open)$$

$$+ \; p(X_1 = is\_closed|U_1 = do\_nothing, X_0 = is\_closed)\, bel(X_0 = is\_closed)$$

$$= 0 \times 0.5 + 1 \times 0.5 = 0.5$$

Bayes Filter - Example 1

$u_1 = do\_nothing \text{ and } z_1 = sense\_open$

Incorporate the measurement

$\overline{bel}(X_1 = is\_open) = 0.5$

$\overline{bel}(X_1 = is\_closed) = 0.5$

$bel(x_1) = \eta\, p(Z_1 = sense\_open |\, x_1)\overline{bel}(x_1)$

For two possible cases, $X_1 = is\_open$ and $X_1 = is\_closed$, we get

$$bel(X_1 = is\_open) = \eta\, p(Z_1 = sense\_open\, |X_1 = is\_open)\, \overline{bel}(X_1 = is\_open)$$
$$= \eta \times 0.6 \times 0.5 = \eta\, 0.3$$

$$bel(X_1 = is\_closed) = \eta\, p(Z_1 = sense\_open\, |X_1 = is\_closed)\, \overline{bel}(X_1 = is\_closed)$$
$$= \eta \times 0.2 \times 0.5 = \eta\, 0.1$$

Normalizing constant: $\eta = (0.3 + 0.1)^{-1} = 2.5$

- $bel(X_1 = is\_closed) = \eta 0.1 = 0.25$
- $bel(X_1 = is\_open) = \eta 0.3 = 0.75$

Better than initial belief at time t=0!

Bayes Filter - Example 1

$$u_2 = push\ and\ z_2 = sense\_open$$

Prediction update:

$$\overline{bel}(X_2 = is\_closed) = 0 \times 0.75 + 0.2 \times 0.25 = 0.05$$

$$\overline{bel}(X_2 = is\_open) = 1 \times 0.75 + 0.8 \times 0.25 = 0.95$$

Measurement Update:

$$bel(X_2 = is\_closed) = \eta \times 0.2 \times 0.05 \simeq 0.017$$

$$bel(X_2 = is\_open) = \eta \times 0.6 \times 0.95 \simeq 0.983$$

Way better than the initial belief at time t=0!

Summary of Bayes Filter

- The robot is modeled as performing a series of alternating measurements and actions
- Given:
  - Sensor model $p(z|x)$
  - Action model $p(x|u, x_{t-1})$
  - Initial Conditions $p(x_0)$
- To compute:
  - Estimate state x of a dynamical system
  - Posterior of the state (Belief):
    $$bel(x_t) = p(x_t|u_1, z_1, \dots, u_t, z_t)$$

1. **Algorithm Bayes_Filter** $(bel(x_{t-1}), u_t, z_t)$:
2. for all x_t do
3. $\overline{bel}(x_t) = \sum_{x_{t-1}} p(x_t|u_t, x_{t-1})\ bel(x_{t-1})$
4. $bel(x_t) = \eta\, p(z_t|x_t)\ \overline{bel}(x_t)$
5. endfor
6. return $bel(x_t)$

Summary of Bayes Filter

- Prediction Step:
  - Incorporate action, which **increases** uncertainty
  - Compute $\overline{bel}(x_t) = p(x_t|z_{1:t-1}, u_{1:t})$
  - Requires Action Model: $p(x_t|u_t, x_{t-1})$
- Measurement/Update Step:
  - Incorporating measurement, which **decreases** uncertainty
  - Compute $bel(x_t) = p(x_t|z_{1:t}, u_{1:t})$
  - Requires Sensor Model: $p(z_t|x_t)$

1. **Algorithm Bayes_Filter** $(bel(x_{t-1}), u_t, z_t)$:
2. for all x_t do
3. $\overline{bel}(x_t) = \sum_{x_{t-1}} p(x_t|u_t, x_{t-1})\ bel(x_{t-1})$
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Fast Robots

Probabilistic Motion Model

$$p(x_t \mid u_t, x_{t-1})$$

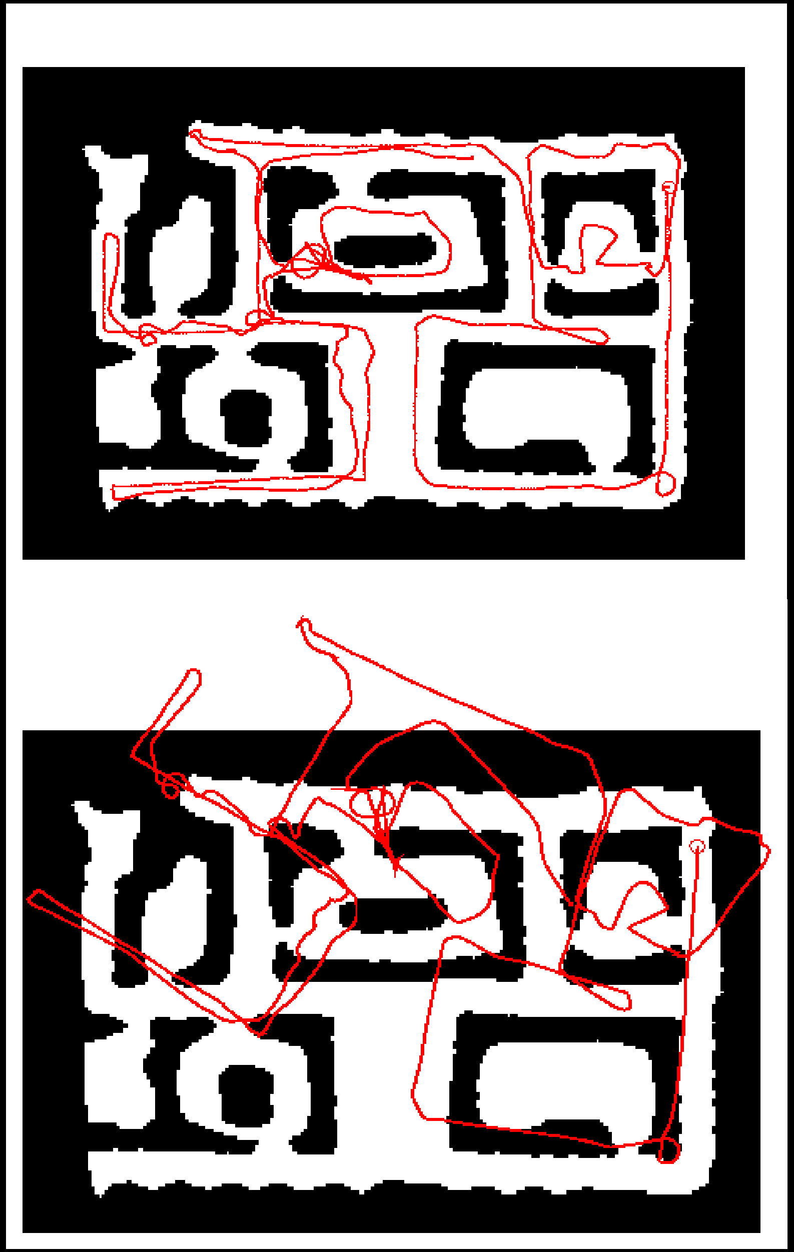
Bayes Filter

1. **Algorithm Bayes_Filter** $(bel(x_{t-1}), u_t, z_t)$:
2. for all x_t do
3. $\overline{bel}(x_t) = \sum_{x_{t-1}} p(x_t|u_t, x_{t-1})\ bel(x_{t-1})$ (Prediction step)
4. $bel(x_t) = \eta\, p(z_t|x_t)\ \overline{bel}(x_t)$
5. endfor
6. return $bel(x_t)$

Transition probability /action model

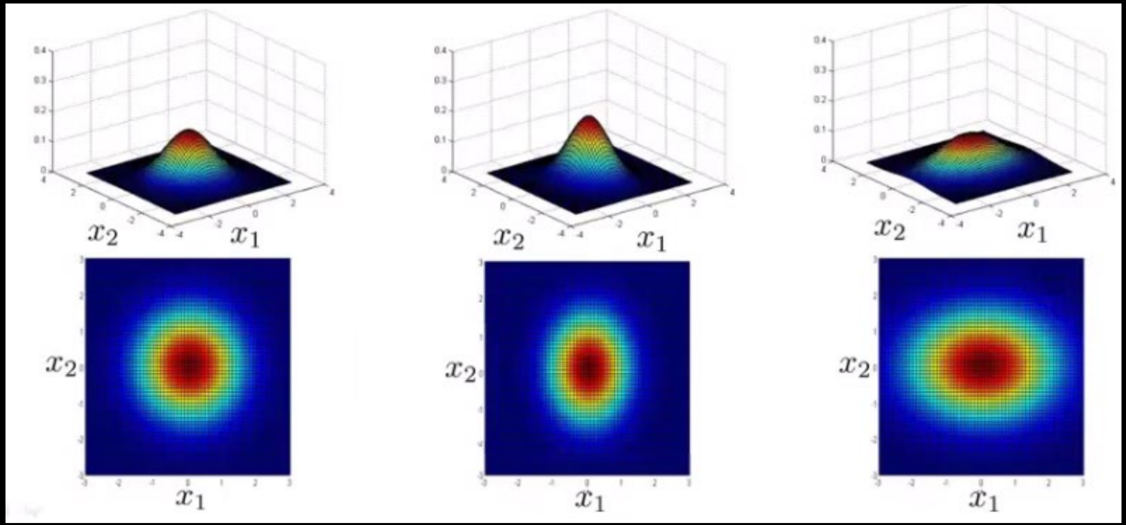
Robot Motion

- Mobile robots on a plane
  - Robot pose $x_t = (x, y, \theta)^T$
- Robot motion is inherently uncertain
  - Transition model: $p(x_t | u_t, x_{t-1})$
- How can we model $p(x_t | u_t, x_{t-1})$ based on kinematic equations?
  - Velocity model
  - Odometry model

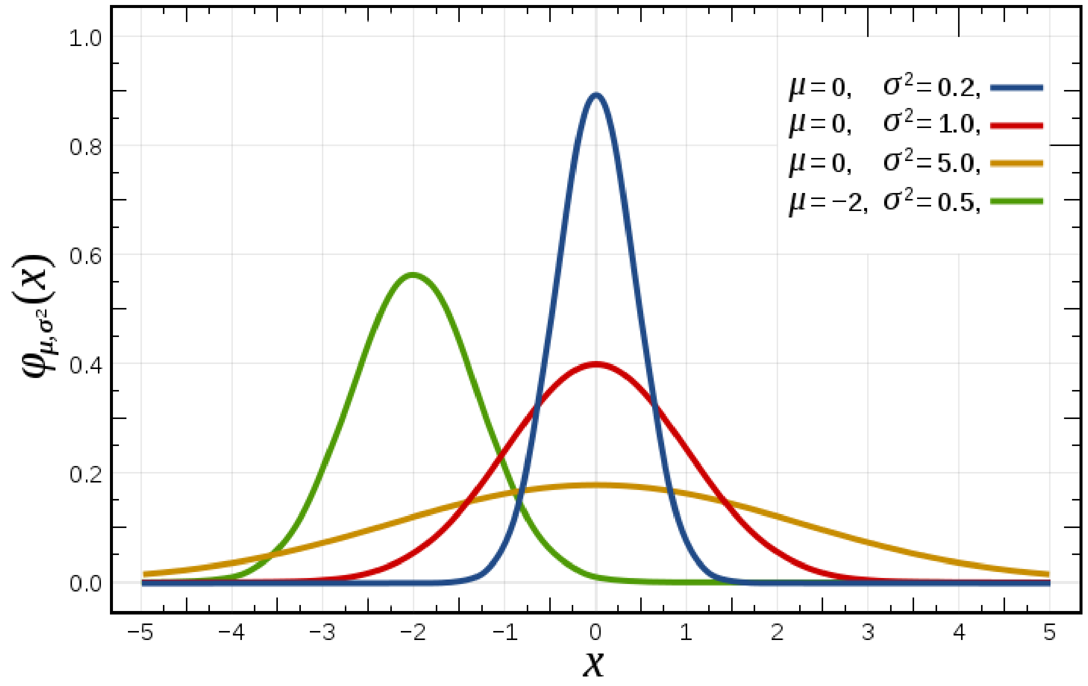


Quick Detour: Probability Distributions

- Gaussian, normal distribution, bell curve
- Defined by two parameters:
  - mean μ
  - standard deviation σ
- Can be defined for multidimensional data



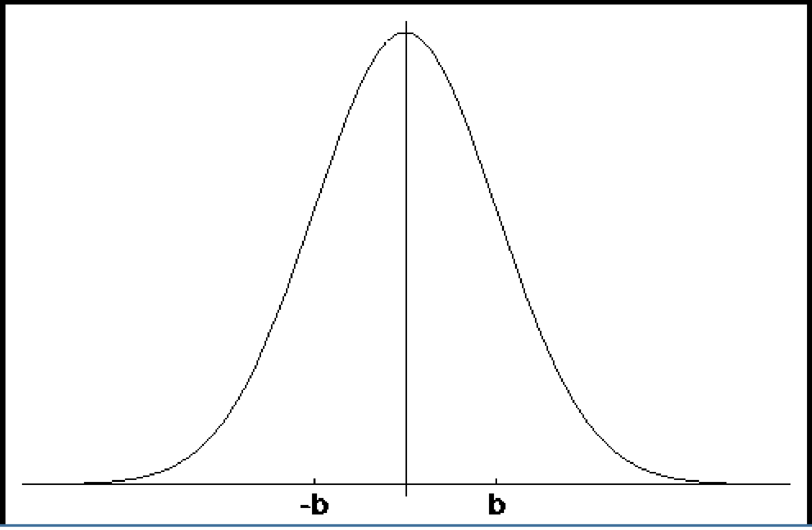
1D Gaussian Probability Density Function



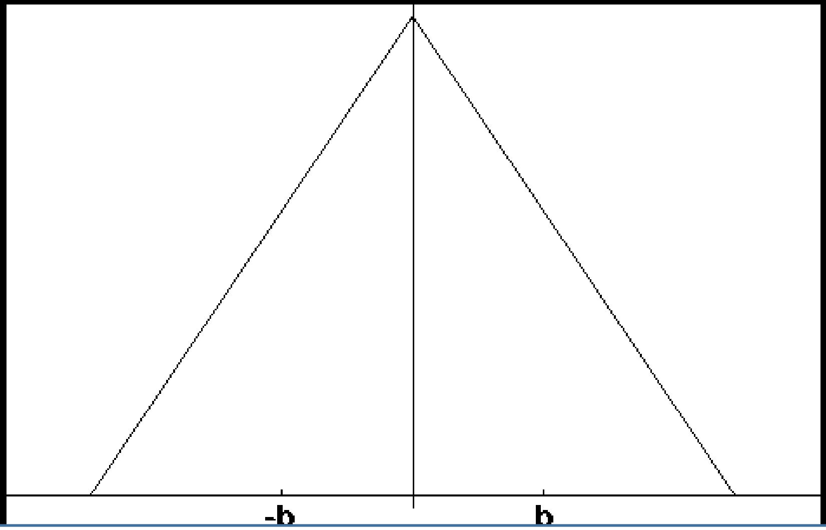
$$f(x \mid \mu, \sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

Quick Detour: Probability Distributions

- 3 inputs: $f(x|\,\mu,\sigma^2)$
- 2 inputs: = $f(x-\mu|0,\sigma^2)$
- Computationally cheaper alternative: triangular distributions



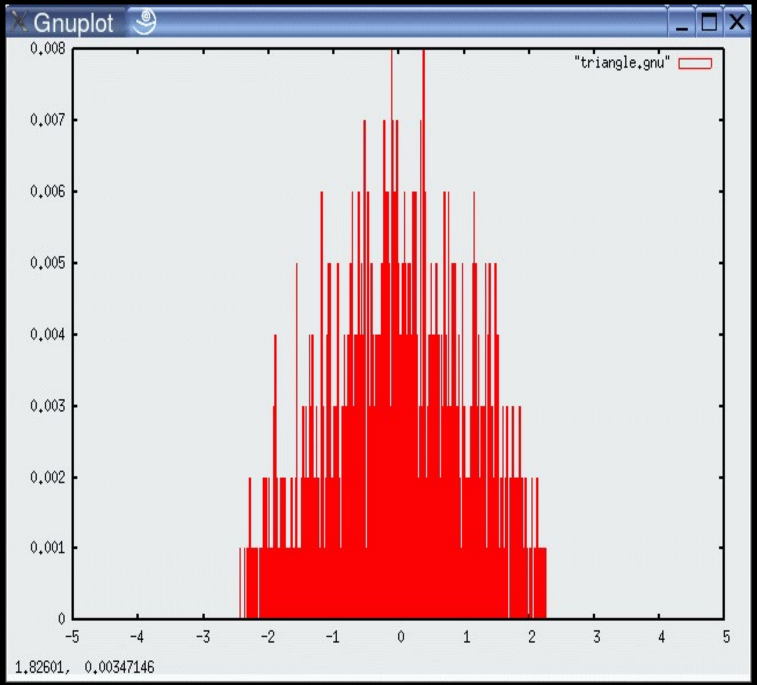
1. **Algorithm prob_normal_distribution**(a, b^2) :
2. `return` $\frac{1}{\sqrt{2\pi b^2}}\, exp\left(\frac{a^2}{2b^2}\right)$

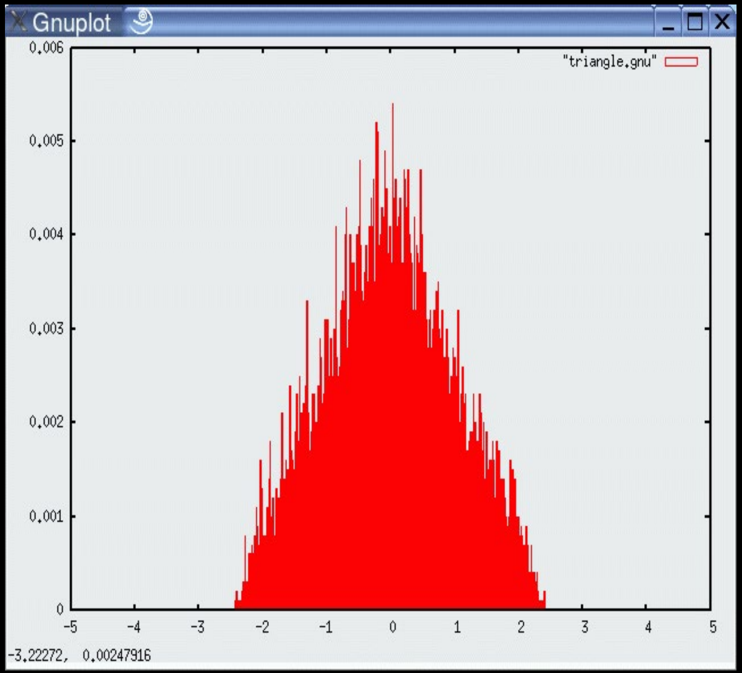


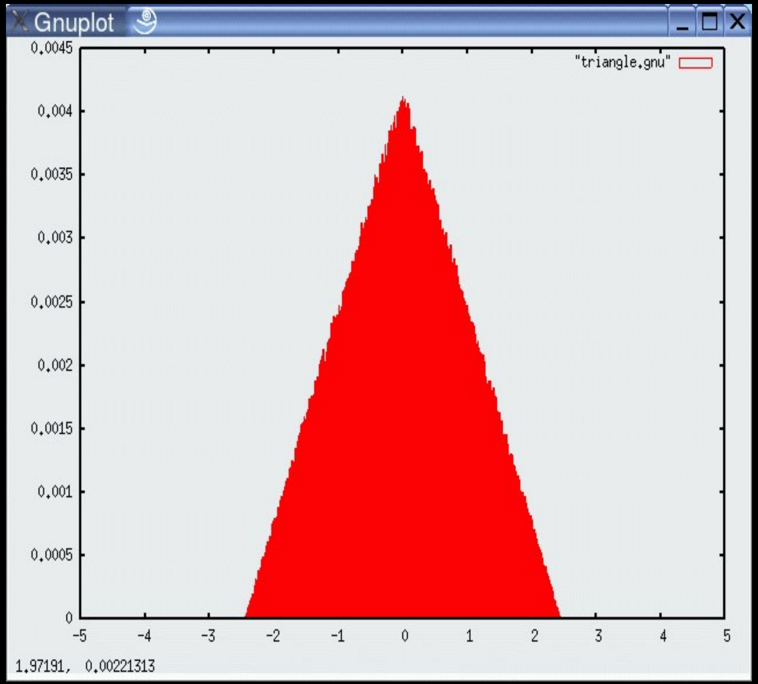
1. **Algorithm prob_triangular_distribution**(a, b^2) :
2. `return` $max\left(0, \frac{1}{\sqrt{6}\;b} - \frac{|a|}{6\;b^2}\right)$

Quick Detour: Probability Distributions

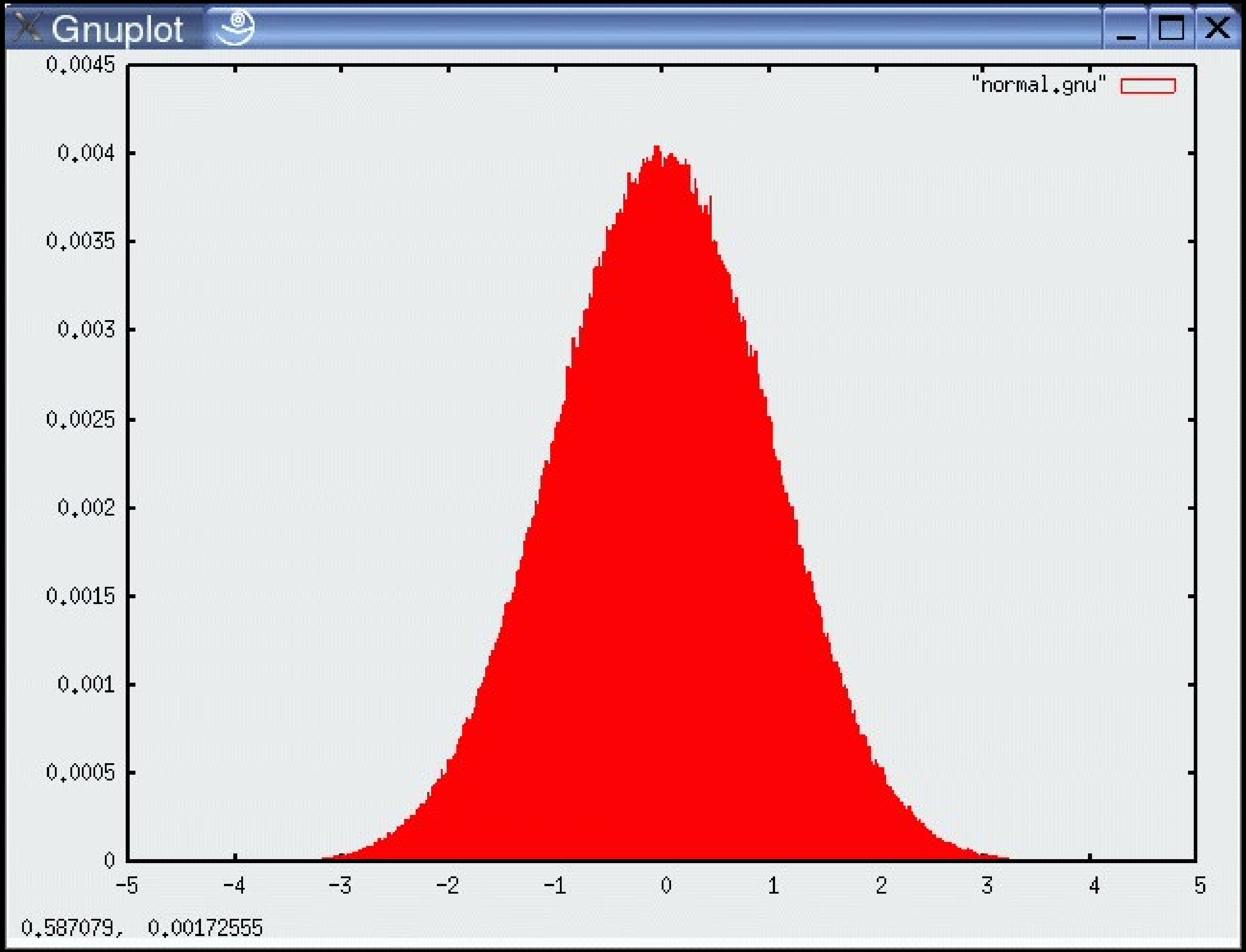
- Sampling algorithms output samples from a given distribution
- Often used to approximate distributions







Quick Detour: Probability Distributions



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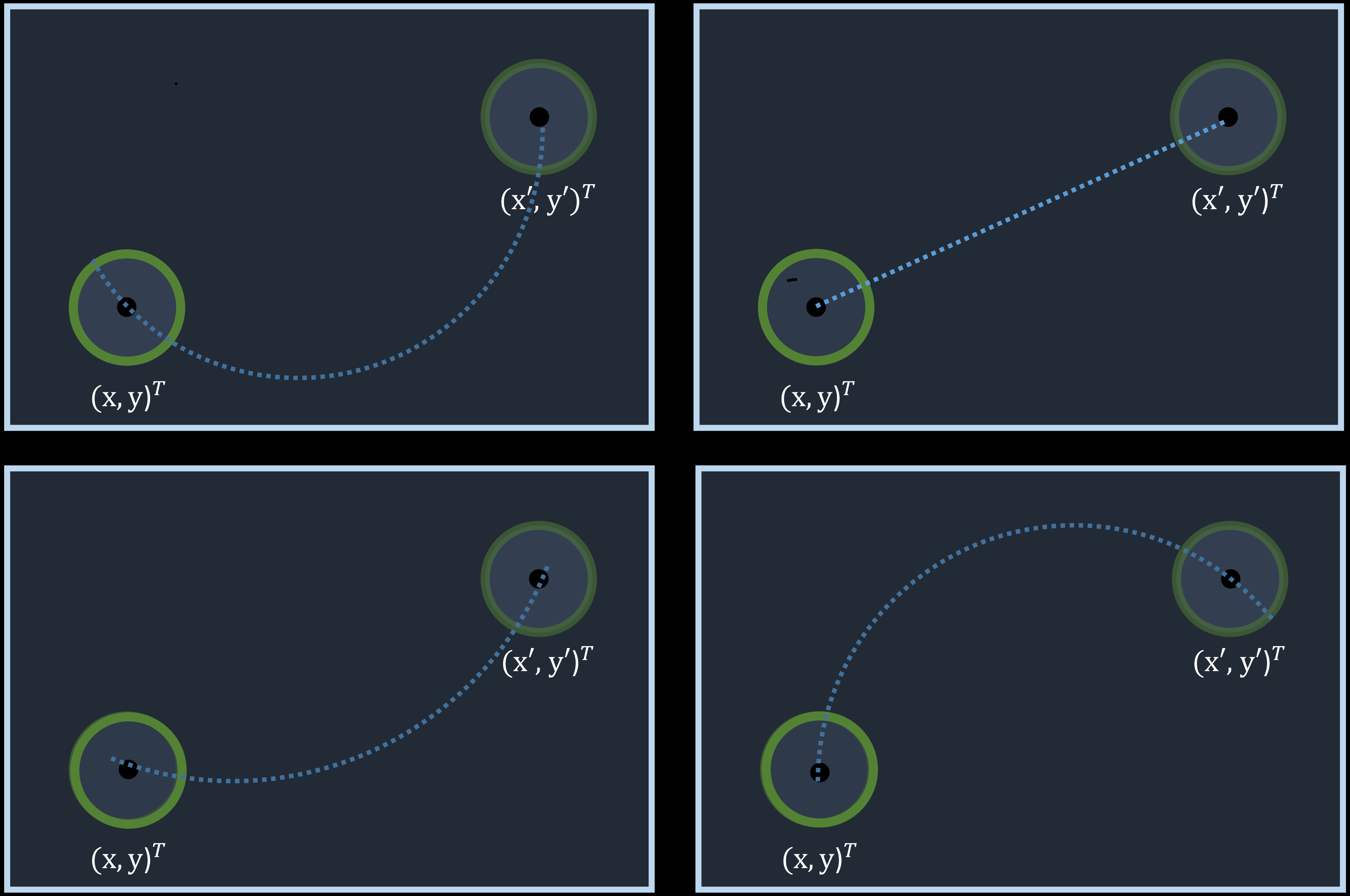
Velocity Model

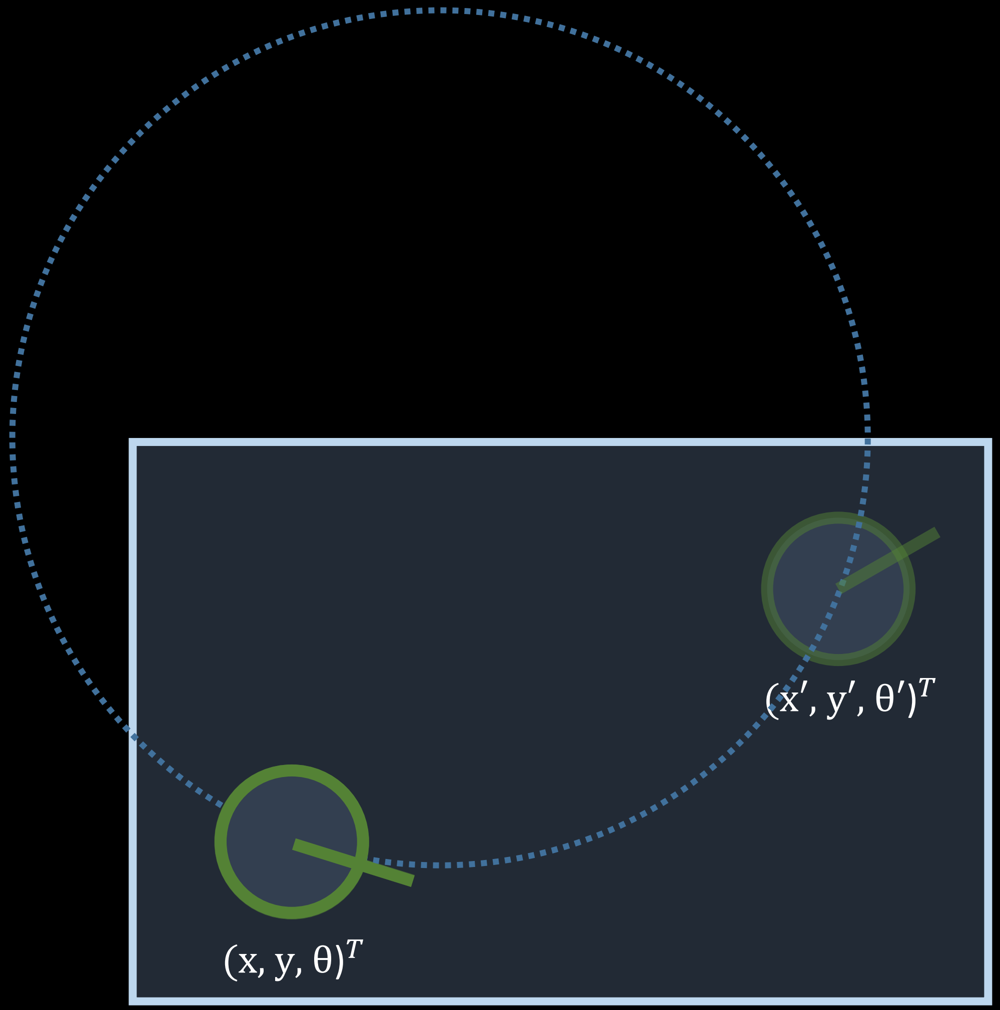
- translation velocity v
- rotational velocity ω
- $u_t = (v, \omega)$

Velocity Model Parameters

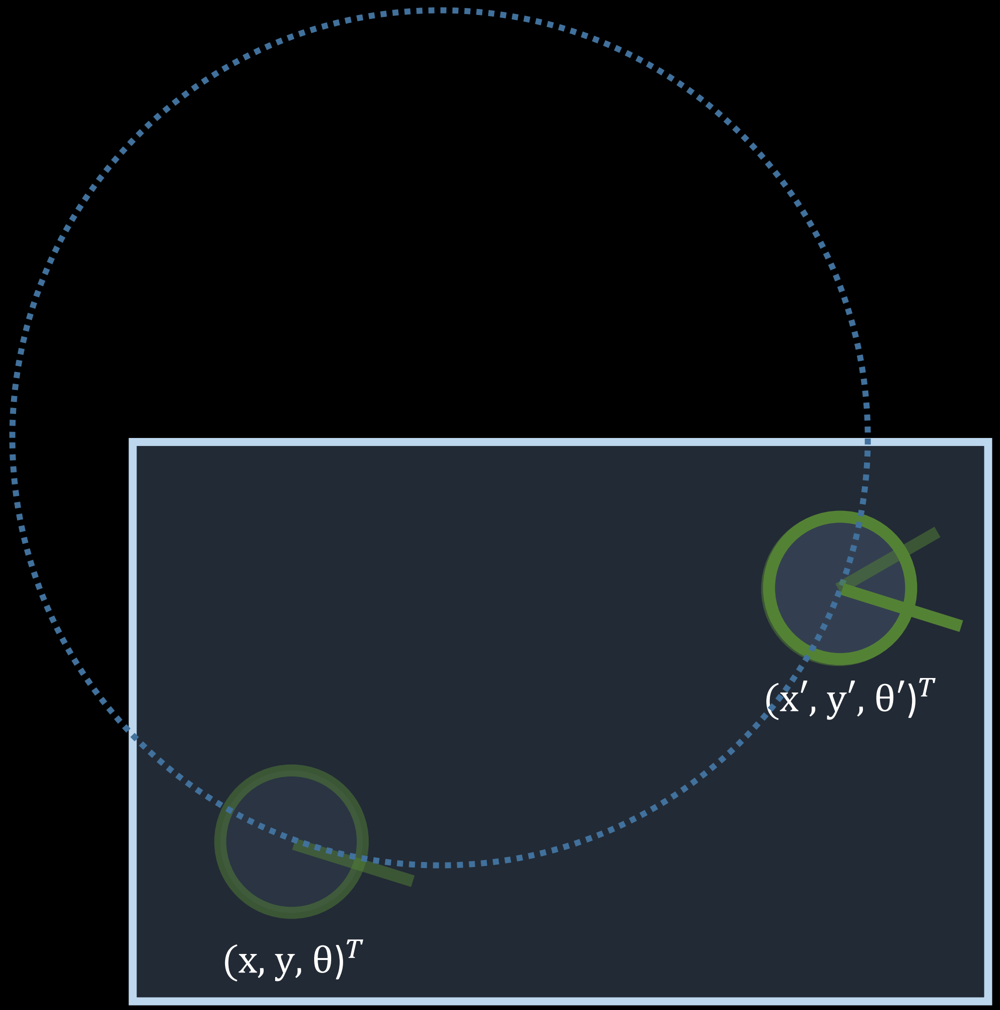
- $u = (v_{right}, v_{left})$
- $u = (v_{COM}, \omega_{COM})$



Velocity Model Parameters



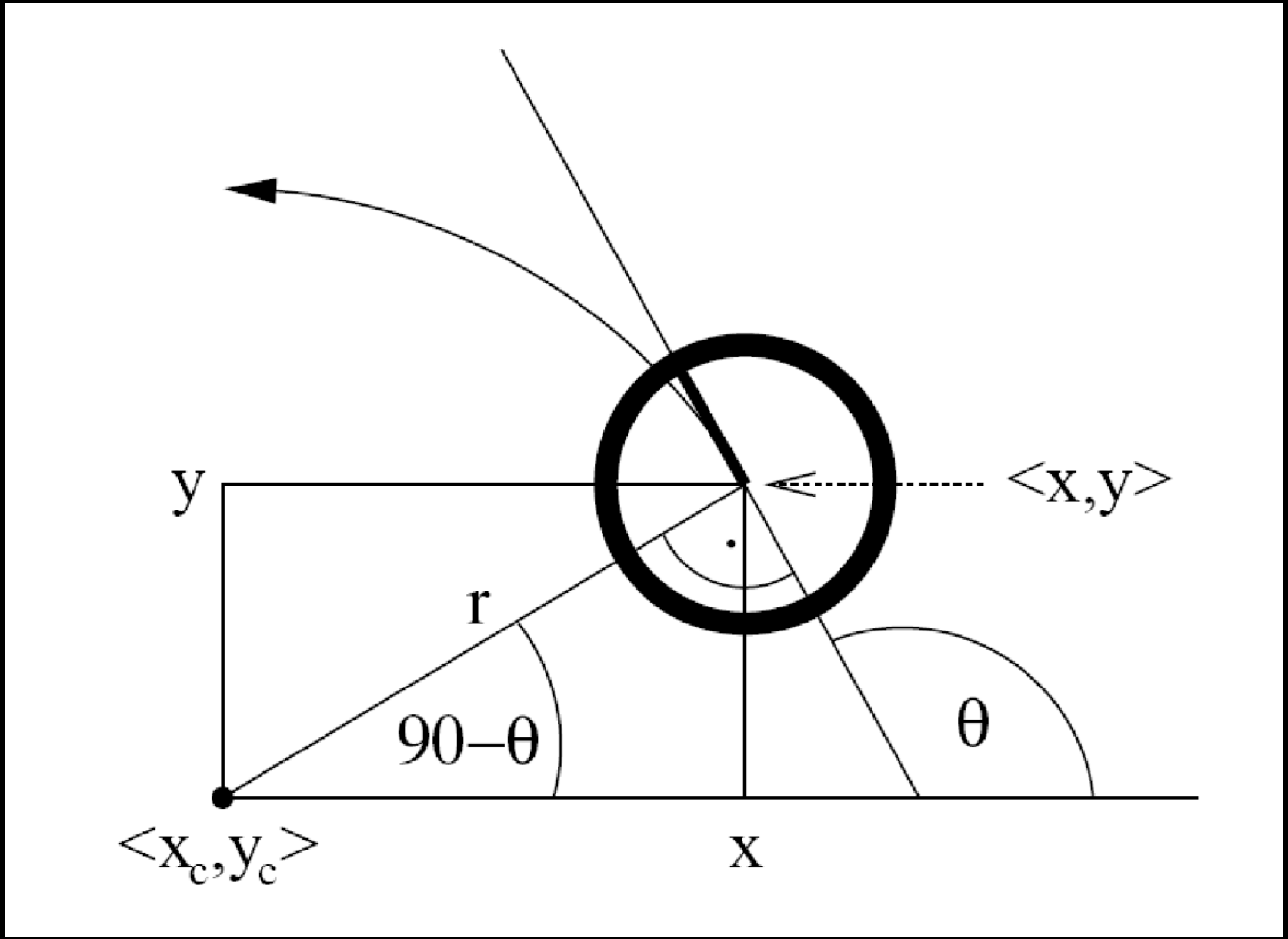
Velocity Model Parameters



(Rotation γ when at new pose)

Velocity Model

- Exact motion: $x_t = (x', y', \theta')^T$
- Start state: $x_{t-1} = (x, y, \theta)^T$
- Control data: $u_t = (v_t,\ \omega_t)^T$
- *(Under the assumption that both velocity components are kept fixed over the time interval)*
- ..and then we add γ



Velocity Model

Algorithm motion_model_velocity(x_t, u_t, x_{t-1}):

$$\mu = \frac{1}{2}\,\frac{(x - x')\cos\theta + (y - y')\sin\theta}{(y - y')\cos\theta - (x - x')\sin\theta}$$

$$x^* = \frac{x + x'}{2} + \mu(y - y')$$

$$y^* = \frac{y + y'}{2} + \mu(x' - x)$$

$$r^* = \sqrt{(x - x^*)^2 + (y - y^*)^2}$$

$$\Delta\theta = \text{atan2}(y' - y^*, x' - x^*) - \text{atan2}(y - y^*, x - x^*)$$

$$\hat{v} = \frac{\Delta\theta}{\Delta t}\, r^*$$

$$\hat{\omega} = \frac{\Delta\theta}{\Delta t}$$

$$\hat{\gamma} = \frac{\theta' - \theta}{\Delta t} - \hat{\omega}$$

Ideal control values

Calculate the error-free control between the states x_{t-1} and x_t

- ***How to add probability?***
  - $f(v_t \mid \hat{v}, \sigma_v^2)$
  - $f(\omega_t \mid \hat{\omega}, \sigma_\omega^2)$
  - $f(\gamma_t \mid \hat{\gamma}, \sigma_\gamma^2)$

Probabilistic Velocity Model

Algorithm motion_model_velocity(x_t, u_t, x_{t-1}):

$$\mu = \frac{1}{2}\frac{(x-x')\cos\theta + (y-y')\sin\theta}{(y-y')\cos\theta - (x-x')\sin\theta}$$

$$x^* = \frac{x+x'}{2} + \mu(y-y')$$

$$y^* = \frac{y+y'}{2} + \mu(x'-x)$$

$$r^* = \sqrt{(x-x^*)^2 + (y-y^*)^2}$$

$$\Delta\theta = \text{atan2}(y'-y^*, x'-x^*) - \text{atan2}(y-y^*, x-x^*)$$

$$\hat{v} = \frac{\Delta\theta}{\Delta t}\, r^*$$

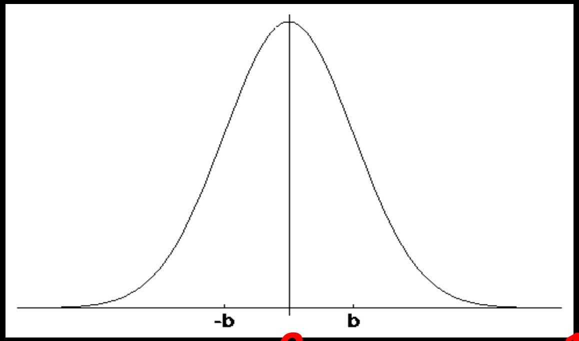
$$\hat{\omega} = \frac{\Delta\theta}{\Delta t}$$

$$\hat{\gamma} = \frac{\theta' - \theta}{\Delta t} - \hat{\omega}$$

$$\textit{return}\ \textbf{prob}(v-\hat{v}, \alpha_1|v| + \alpha_2|\omega|) \cdot \textbf{prob}(\omega-\hat{\omega}, \alpha_3|v| + \alpha_4|\omega|) \cdot \textbf{prob}(\hat{\gamma}, \alpha_5|v| + \alpha_6|\omega|)$$

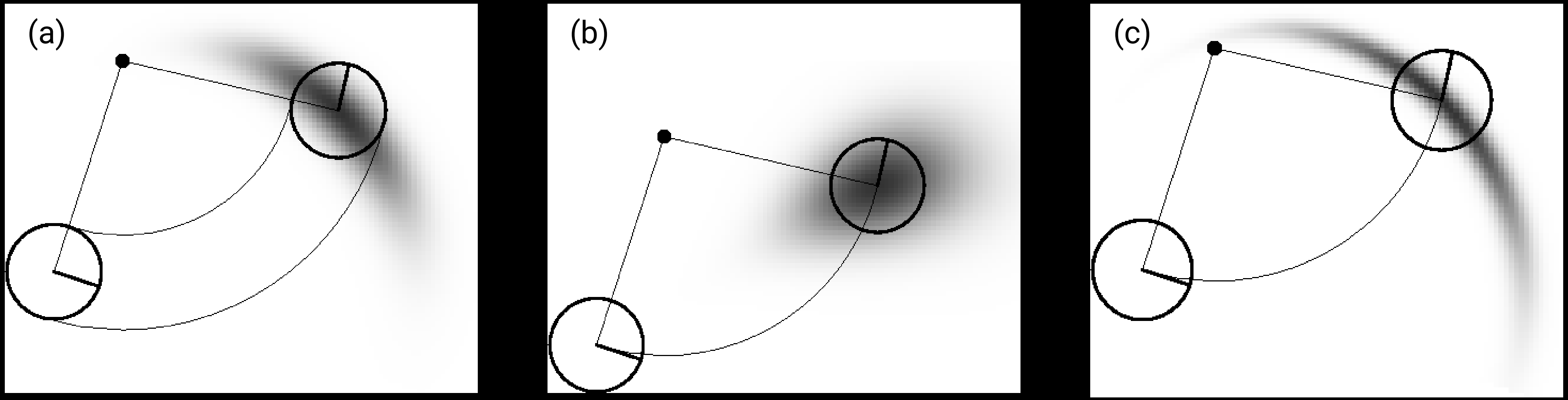
Calculate the error-free control between the states x_{t-1} and x_t

- ***How to add probability?***
  - $f(v_t | \hat{v}, \sigma_v^2)$
    - $f(v_t - \hat{v} | 0, \sigma_v^2)$
  - $f(\omega_t | \hat{\omega}, \sigma_\omega^2)$
  - $f(\gamma_t | \hat{\gamma}, \sigma_\gamma^2)$



0 **100**

Velocity Motion Model

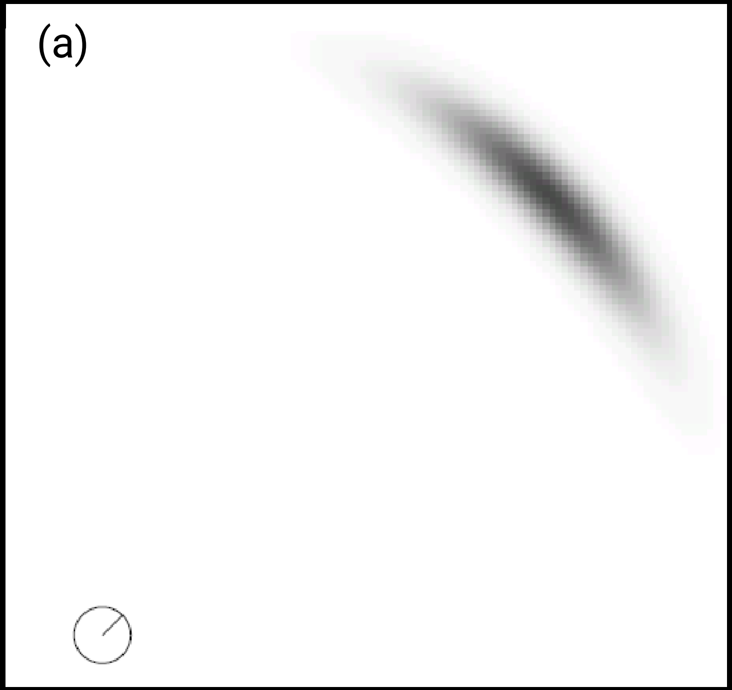


(darker regions are more probable)

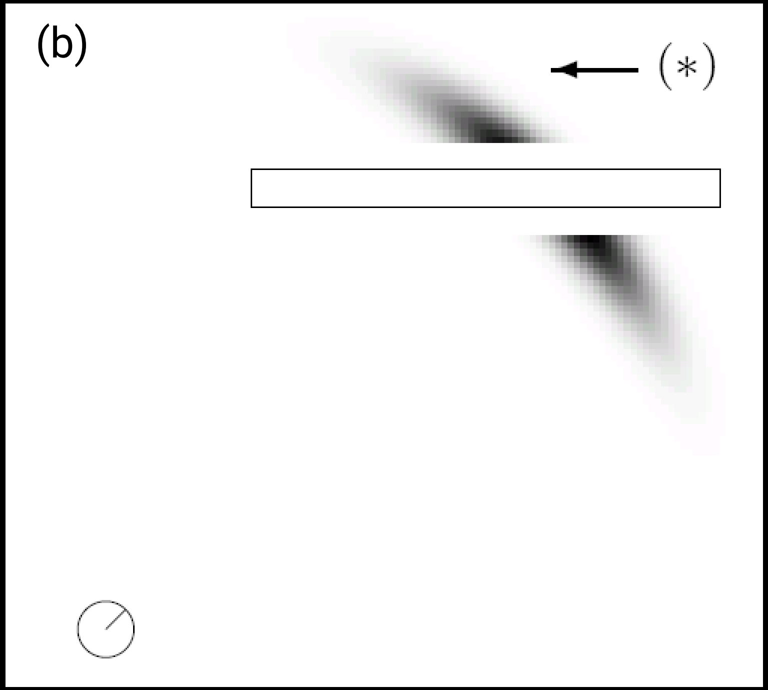
The velocity motion model for different noise parameters settings for the same control $u_t = (v_t,\ \omega_t)^T$ projected in the x-y space

a) More angular than translational noise
b) Larger transitional noise
c) Large angular and translational noise parameters

Velocity Model with a Map



$p(x_t \mid u_t, x_{t-1})$



$p(x_t \mid u_t, x_{t-1}, map)$

Sampling from Velocity Model

Algorithm sample_motion_model_velocity(u_t, x_{t-1})**:**

$\hat{v} = v + \mathbf{sample}(\alpha_1|v| + \alpha_2|\omega|)$

$\hat{\omega} = \omega + \mathbf{sample}(\alpha_3|v| + \alpha_4|\omega|)$

$\hat{\gamma} = \mathbf{sample}(\alpha_5|v| + \alpha_6|\omega|)$

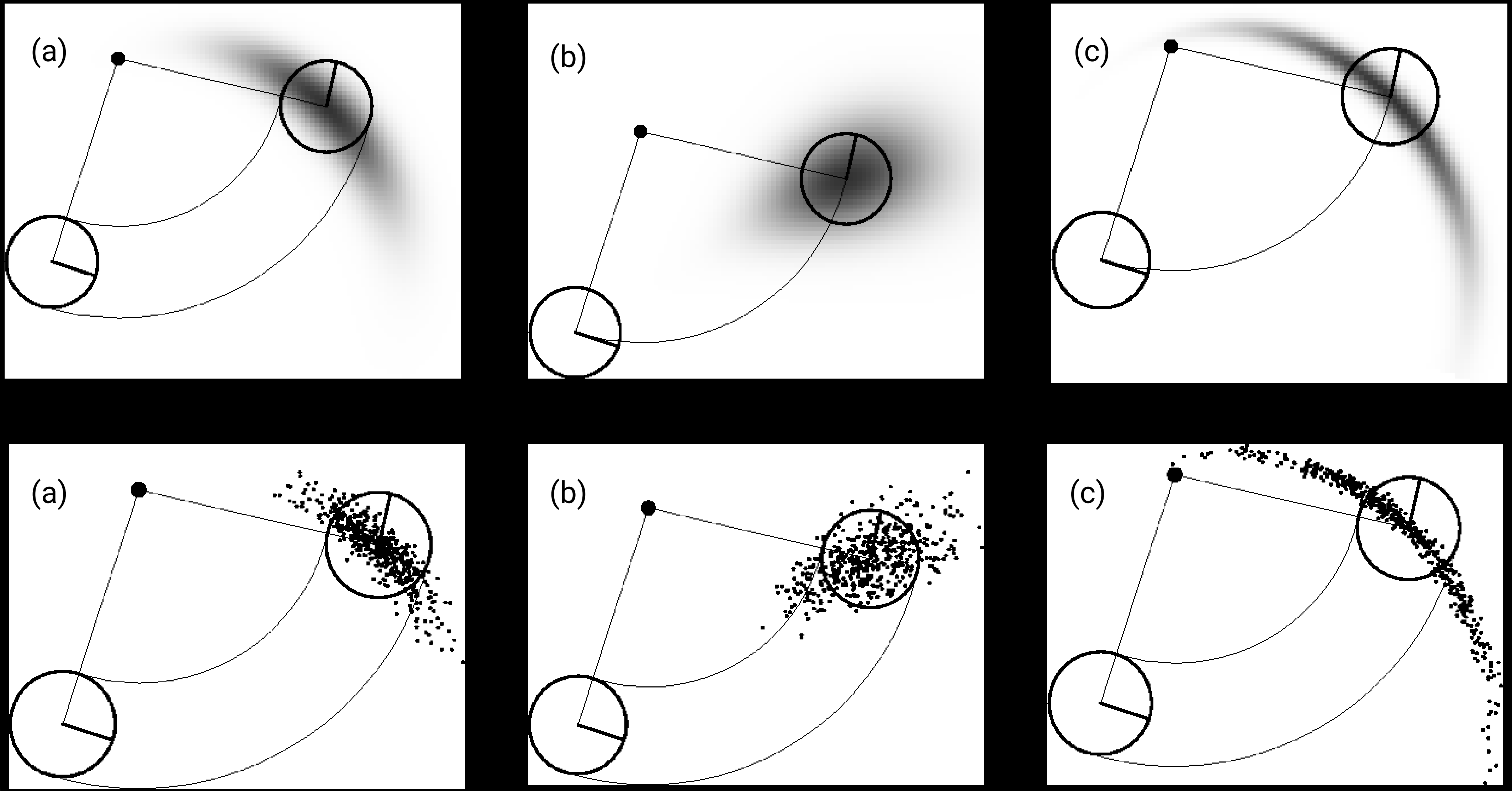
$x' = x - \frac{\hat{v}}{\hat{\omega}} \sin\theta + \frac{\hat{v}}{\hat{\omega}}\ sin(\theta + \hat{\omega}\Delta t)$

$y' = y + \frac{\hat{v}}{\hat{\omega}} \cos\theta - \frac{\hat{v}}{\hat{\omega}}\ cos(\theta + \hat{\omega}\Delta t)$

$\theta' = \theta + \hat{\omega}\Delta t + \hat{\gamma}\Delta t$

return $x_t = (x', y', \theta')^T$

Sampling from Velocity Model

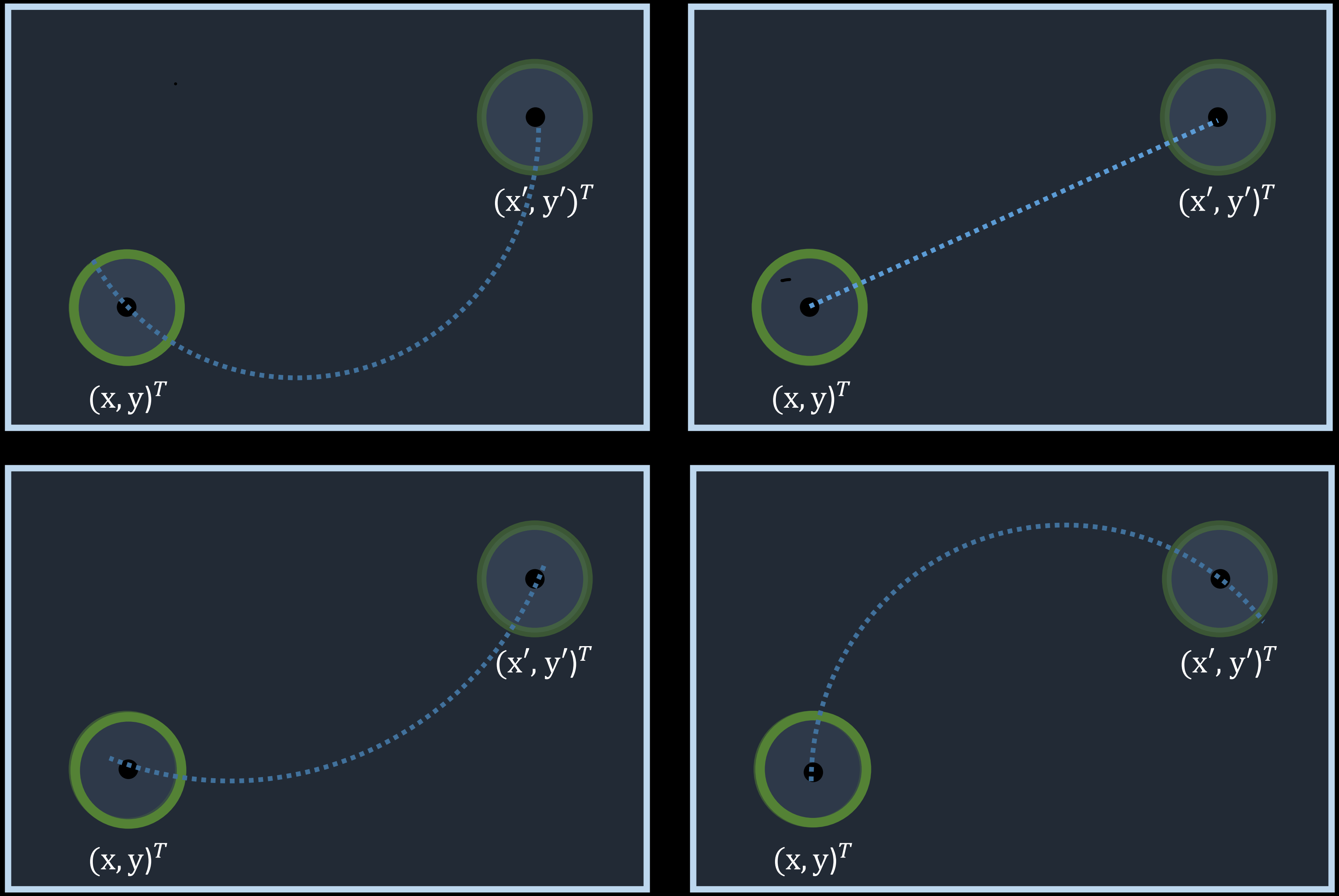


Velocity Model Parameters

- $u = (v_{right}, v_{left})$
- $u = (v_{COM}, \omega_{COM})$

- How would you use this in your system?

- Pros
  - Prediction/planning
- Cons
  - Parameter tuning
  - Inaccurate



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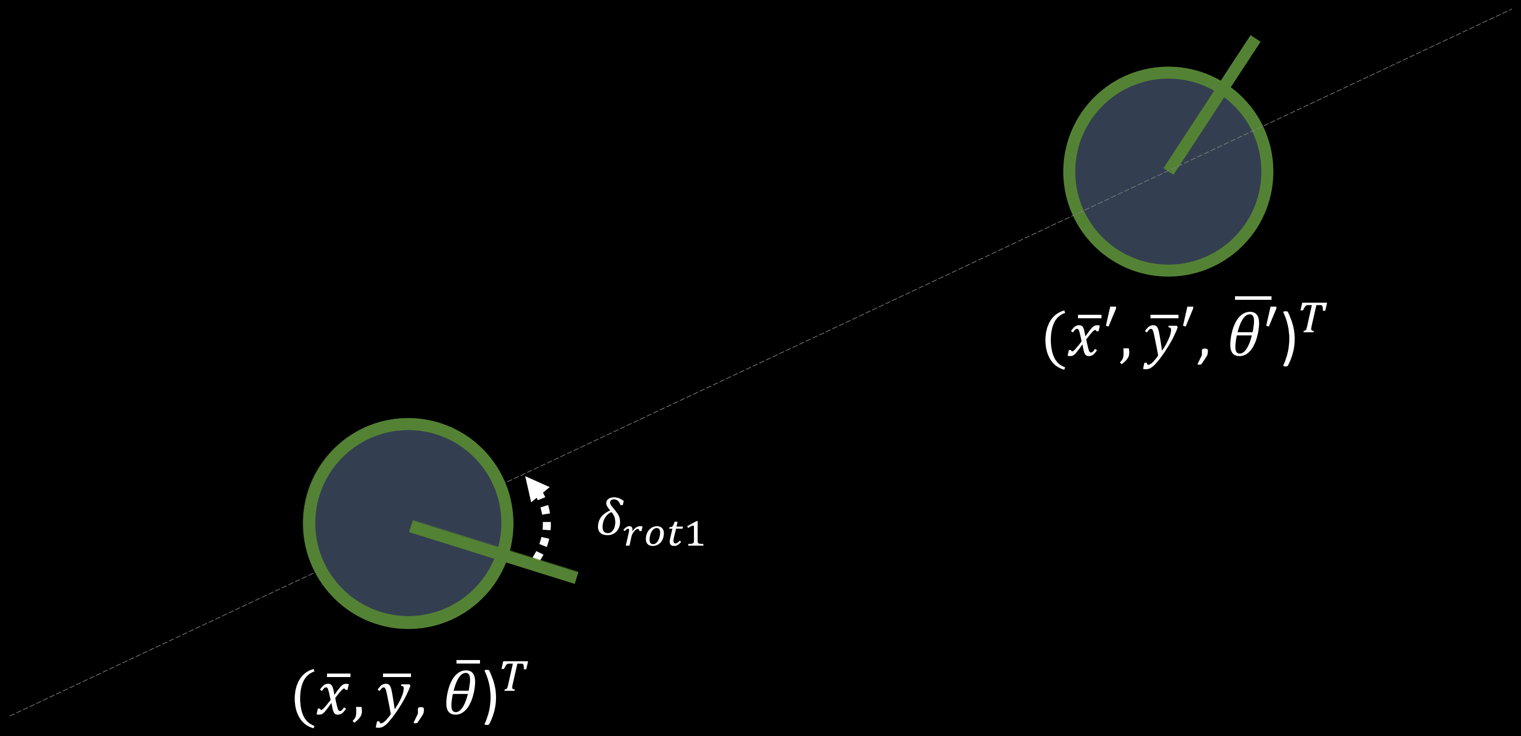
Odometry Model

$$u_t = (\overline{x_{t-1}}, \bar{x}_t)^T$$

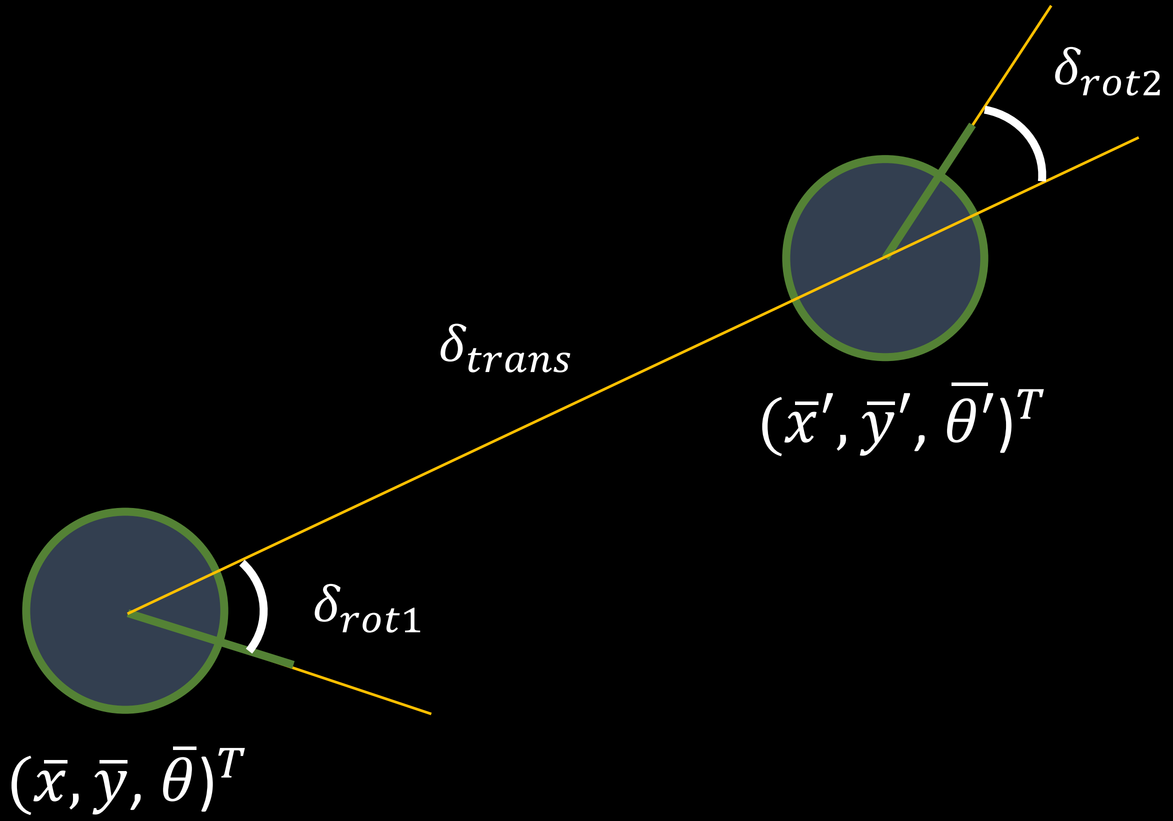
Odometry Model Parameters



Odometry Model Parameters

- Relative odometry motion is transformed into a sequence of three steps
  - Initial rotation δ_{rot1}
  - Translation δ_{trans}
  - Final Rotation δ_{rot2}
- These three parameters are sufficient to reconstruct the relative motion between two robot states

$$u_t = (\delta_{rot1}, \delta_{trans}, \delta_{rot2})^T$$

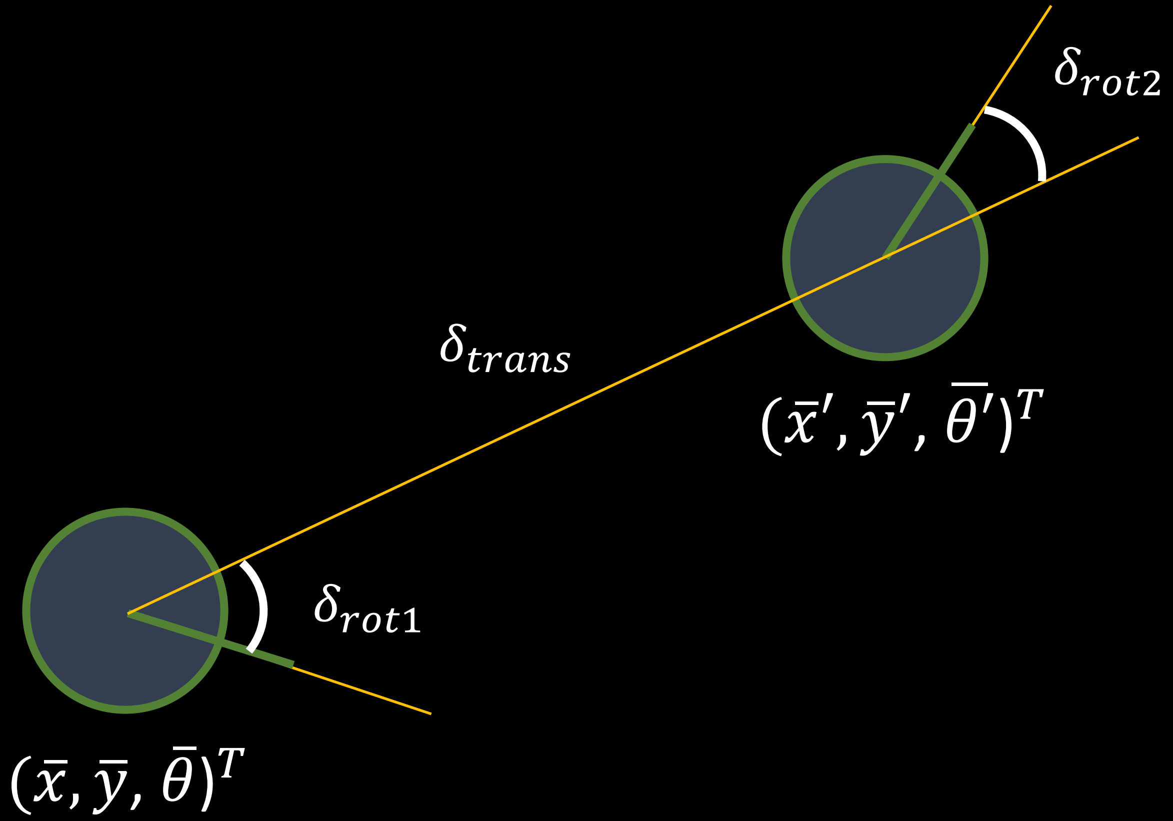


Odometry Model Parameters

$$\delta_{rot1} = \text{atan2}(\bar{y}' - \bar{y}, \bar{x}' - \bar{x}) - \bar{\theta}$$

$$\delta_{trans} = \sqrt{(\bar{y}' - \bar{y})^2 + (\bar{x}' - \bar{x})^2}$$

$$\delta_{rot2} = \bar{\theta}' - \bar{\theta} - \delta_{rot1}$$



Algorithm motion_model_odometry(x_t, u_t, x_{t-1}) :

$\delta_{rot1} = \texttt{atan2}(\bar{y}' - \bar{y}, \bar{x}' - \bar{x}) - \bar{\theta}$

$\delta_{trans} = \sqrt{(\bar{x}' - \bar{x})^2 + (\bar{y}' - \bar{y})^2}$

$\delta_{rot2} = \bar{\theta}' - \bar{\theta} - \delta_{rot1}$

$\hat{\delta}_{rot1} = \texttt{atan2}(y' - y, x' - x) - \theta$

$\hat{\delta}_{trans} = \sqrt{(x' - x)^2 + (y' - y)^2}$

$\hat{\delta}_{rot2} = \theta' - \theta - \hat{\delta}_{rot1}$

$p_1 = \mathbf{prob}(\delta_{rot1} - \hat{\delta}_{rot1}, \alpha_1 \hat{\delta}^2_{rot1} + \alpha_2 \hat{\delta}^2_{trans})$

$p_2 = \mathbf{prob}(\delta_{trans} - \hat{\delta}_{trans}, \alpha_3 \hat{\delta}^2_{trans} + \alpha_4 \hat{\delta}^2_{rot1} + \alpha_4 \hat{\delta}^2_{rot2})$

$p_3 = \mathbf{prob}(\delta_{rot2} - \hat{\delta}_{rot2}, \alpha_1 \hat{\delta}^2_{rot2} + \alpha_2 \hat{\delta}^2_{trans})$

return $p_1.p_2.p_3$

Calculate the relative motion parameters from odometry readings
(what the robot did)

Calculate the relative motion parameters for the given states x_{t-1} and x_t
(what the robot did ideally)

Algorithm sample_motion_model_odometry(x_{t-1}, u_t) :

$$\delta_{rot1} = \mathtt{atan2}(\bar{y}' - \bar{y}, \bar{x}' - \bar{x}) - \bar{\theta}$$

$$\delta_{trans} = \sqrt{(\bar{x}' - \bar{x})^2 + (\bar{y}' - \bar{y})^2}$$

$$\delta_{rot2} = \bar{\theta}' - \bar{\theta} - \delta_{rot1}$$

Calculate the relative motion parameters from odometry readings

$$\hat{\delta}_{rot1} = \delta_{rot1} - \textbf{sample}(\alpha_1 \hat{\delta}^2_{rot1} + \alpha_2 \hat{\delta}^2_{trans})$$

$$\hat{\delta}_{trans} = \delta_{rot1} - \textbf{sample}(\alpha_3 \hat{\delta}^2_{trans} + \alpha_4 \hat{\delta}^2_{rot1} + \alpha_4 \hat{\delta}^2_{rot2})$$

$$\hat{\delta}_{rot2} = \delta_{rot1} - \textbf{sample}(\alpha_1 \hat{\delta}^2_{rot2} + \alpha_2 \hat{\delta}^2_{trans})$$

Add noise to calculated motion parameters

$$x' = x + \hat{\delta}_{trans} \cos(\theta + \hat{\delta}_{rot1})$$

$$y' = y + \hat{\delta}_{trans} \sin(\theta + \hat{\delta}_{rot1})$$

$$\theta' = \theta + \hat{\delta}_{rot1} + \hat{\delta}_{rot2}$$

$$\mathtt{return}\ \ x_t = (x', y', \theta')^T$$

Calculate the sample state

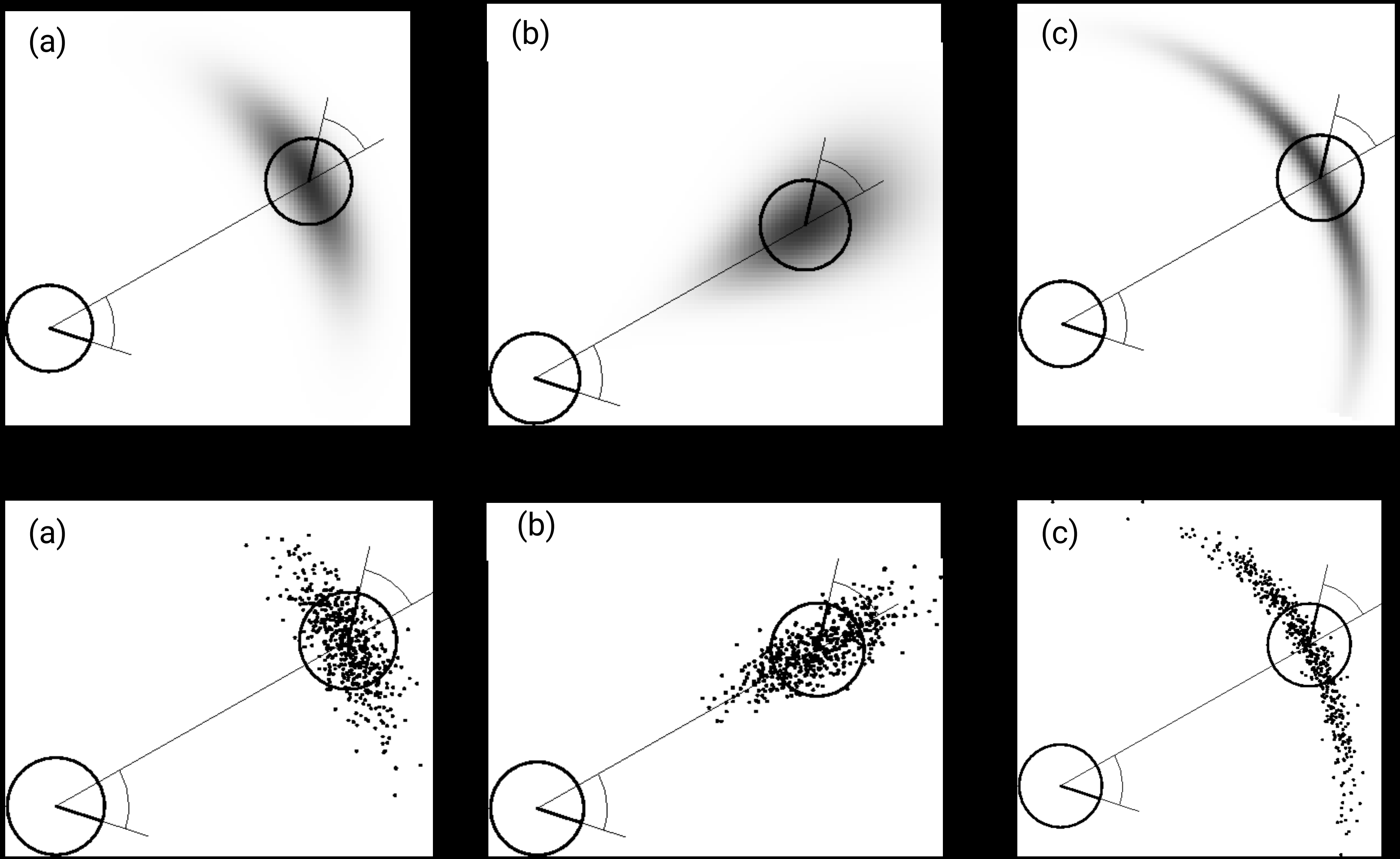
Odometry Model

- $u_t = (\overline{x_{t-1}}, \bar{x}_t)^T$

- How would you use this model in your system?

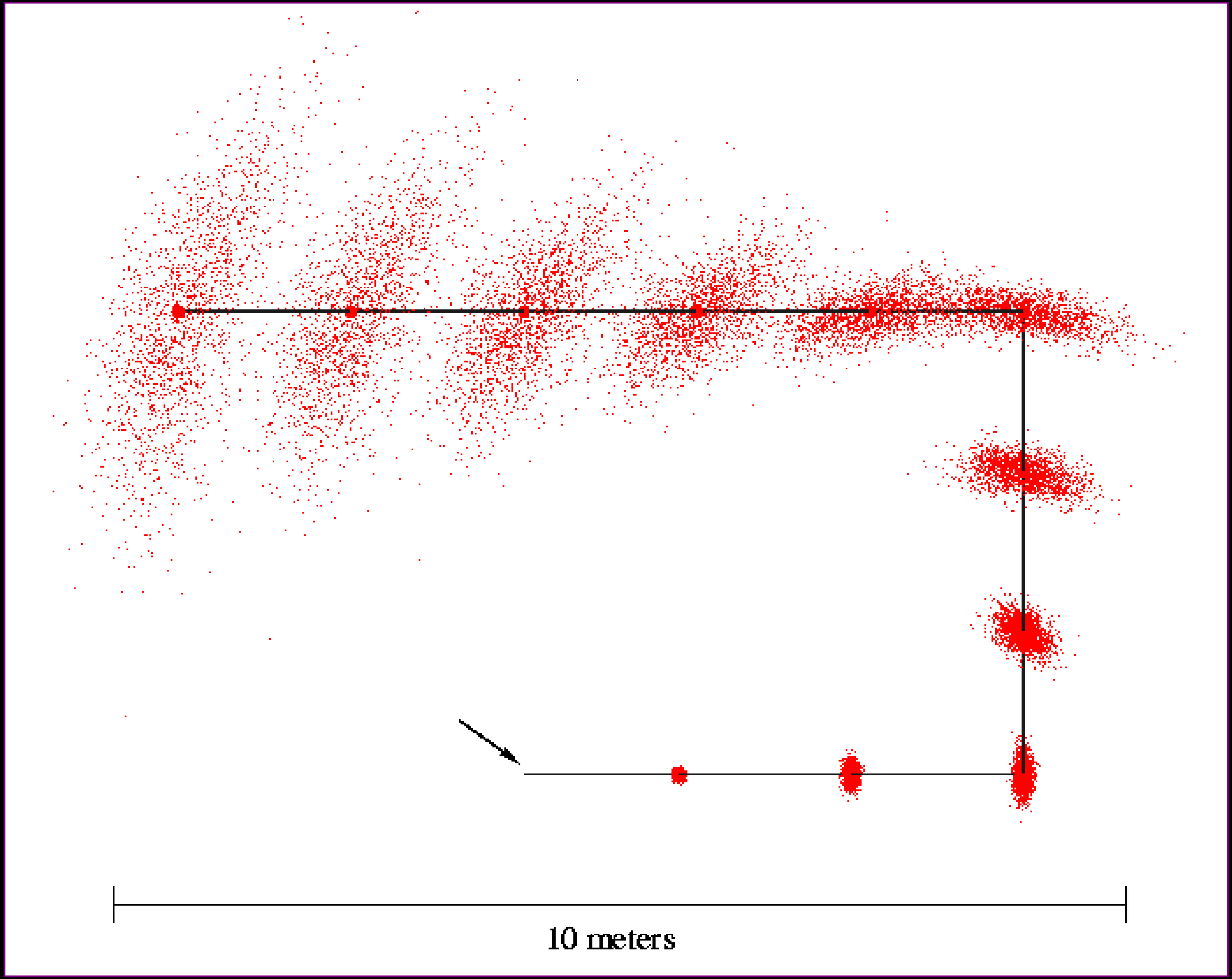
- Odometry is available *after the robot has moved*
  - Can be used for estimation algorithms (e.g. localization and mapping)
  - Cannot be used for prediction (e.g. probabilistic motion planning)

Sampling from odometry Model



Sampling from the odometry model, using the same error parameters as in the previous slides with 500 samples in each.

Repeated Sampling from our odometry motion model



Reference

1. Thrun, Sebastian, Wolfram Burgard, and Dieter Fox. Probabilistic robotics. MIT press, 2005.
2. http://ais.informatik.uni-freiburg.de/teaching/ss11/robotics/slides/06-motion-models.pdf