ECE 4160/5160
MAE 4910/5910

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Fast Robots

Bayes Filter II
Examples

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Fast Robots

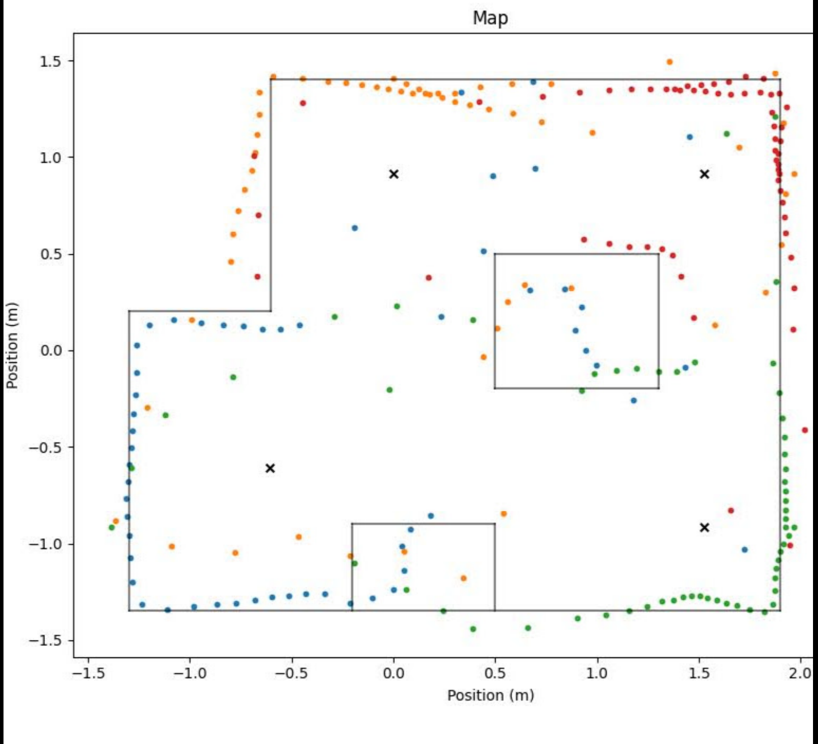
LAB 9

https://cei-lab.github.io/FastRobots-2023/Lab9.html

LAB 9 - Mapping

- Objective: Generate map using your robot and ToF sensor
- Strategy: Place your robot in (at least) 4 marked positions on the floor and spin while taking measurements.
- Control:
  - Open loop
  - Orientation control
  - Angular speed control
- Sanity check: Polar plot, repeated polar plots
- Scatter plot: Use the transformation matrix
- Convert to a line-based map

- Great example from 2022
  https://pages.github.coecis.cornell.edu/avp34/ECE4600-webpage/lab9.html



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Fast Robots

Bayes Filter II
Examples

Bayes Filter II

Algorithm Bayes_Filter $(bel(x_{t-1}), u_t, z_t)$:

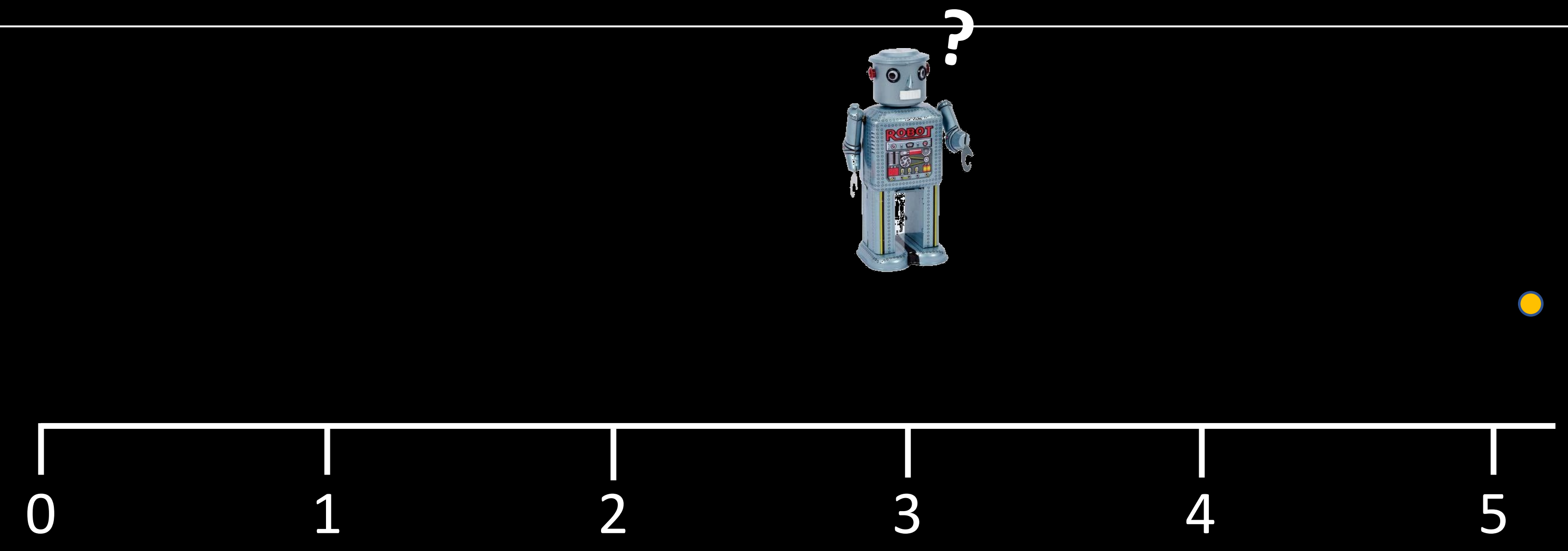
1. for all x_t do
2. $\overline{bel}(x_t) = \sum_{x_{t-1}} p(x_t|u_t, x_{t-1})\; bel(x_{t-1})$ [Prediction Step]
3. $bel(x_t) = \eta\, p(z_t|x_t)\; \overline{bel}(x_t)$ [Update/Measurement Step]
4. endfor
5. return $bel(x_t)$

- Example 1
  - Robot in a 1D world
  - The importance of having some belief in all states
- Example 2
  - Bayes with beans
  - Remember to normalize!
- Example 3
  - (x,y)-robot in a grid world
  - Computational efficiency
    - Matrices
    - Pre-cache observations

Bayes Filter - Example 1

- ***What do we need to run the Bayes filter?***

$p(x|z) = ?$

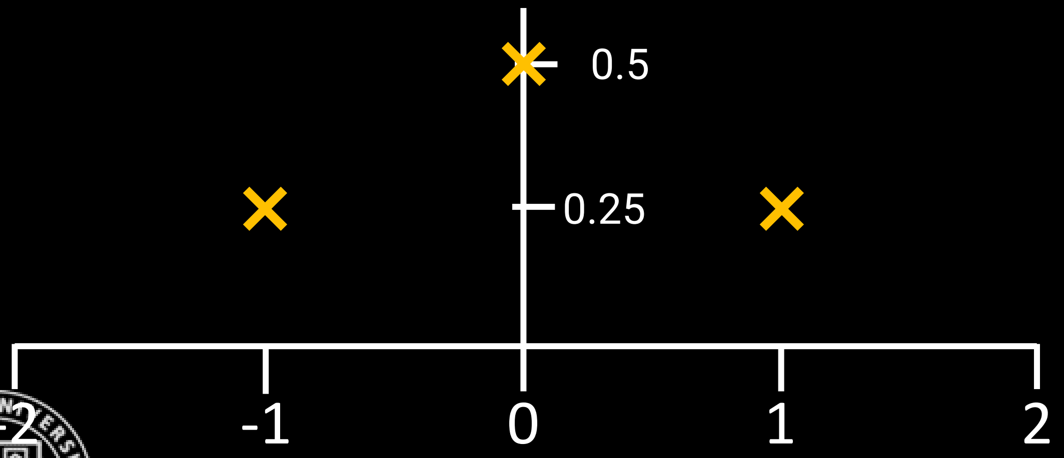


Map

$P(Z{=}door \mid X{=}5) = 0.5$

$P(Z{=}door \mid X{=}4) = 0.25$

$P(Z{=}door \mid X{=}3) = 0$



Sensor Model

$p(x{+}1 \mid x, u{=}{+}1) = 0.5$

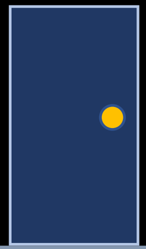
$p(x \mid x, u{=}{+}1) = 0.5$

$p(x{-}1 \mid x, u{=}{-}1) = 0.5$

$p(x \mid x, u{=}{-}1) = 0.5$

Motion Model

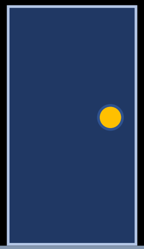
Bayes Filter - Example 1



At $t = 0$, no information

State	0	1	2	3	4	5
$p(x_0)$						

Bayes Filter - Example 1



At $t = 0$, no information

State	0	1	2	3	4	5
$p(x_0)$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

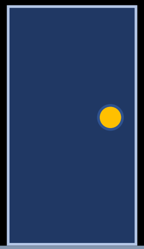
At $t = 1$, $U_1 = \text{do\_nothing}$, $Z_1 = \text{door}$

State	0	1	2	3	4	5
$p(x_1)$						

Do we have to do the prediction step?

Do the update step!

Bayes Filter - Example 1



At $t = 0$, no information

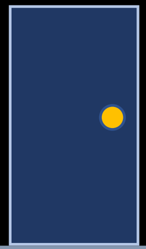
State	0	1	2	3	4	5
$p(x_0)$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

At $t = 1$, $U_1 = \text{do\_nothing}$, $Z_1 = \text{door}$

State	0	1	2	3	4	5
$p(x_1)$	0	0	0	0	$\frac{\frac{1}{6}\times\frac{1}{4}}{\frac{1}{6}\times\frac{1}{4}+\frac{1}{6}\times\frac{1}{2}}$	$\frac{\frac{1}{6}\times\frac{1}{2}}{\frac{1}{6}\times\frac{1}{4}+\frac{1}{6}\times\frac{1}{2}}$

State	0	1	2	3	4	5
$p(x_1)$	0	0	0	0	$\frac{1}{3}$	$\frac{2}{3}$

Bayes Filter - Example 1



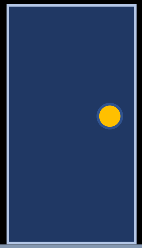
At $t = 1$, $U_1 = \text{do\_nothing}$, $Z_1 = \text{door}$

State	0	1	2	3	4	5
$p(x_1)$	0	0	0	0	$\frac{1}{3}$	$\frac{2}{3}$

At $t = 2$, $U_2 = -1$

State	0	1	2	3	4	5
$p(x_2)$						

Bayes Filter - Example 1



At $t = 1$, $U_1 = \text{do\_nothing}$, $Z_1 = \text{door}$

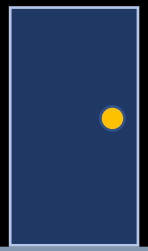
State	0	1	2	3	4	5
$p(x_1)$	0	0	0	0	$\frac{1}{3}$	$\frac{2}{3}$

At $t = 2$, $U_2 = -1$

State	0	1	2	3	4	5
$p(x_2)$	0	0	0	$\frac{1}{3} \times \frac{1}{2}$	$\frac{1}{3} \times \frac{1}{2} + \frac{2}{3} \times \frac{1}{2}$	$\frac{2}{3} \times \frac{1}{2}$

State	0	1	2	3	4	5
$p(x_2)$	0	0	0	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{1}{3}$

Bayes Filter - Example 1



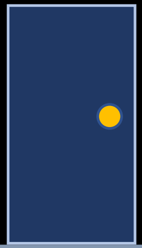
At $t = 2$, $U_2 = -1$

State	0	1	2	3	4	5
$p(x_1)$	0	0	0	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{1}{3}$

At $t = 2$, $U_2 = -1$, $Z_2 = \text{door}$

State	0	1	2	3	4	5
$p(x_2)$						

Bayes Filter - Example 1



At $t = 1$, $U_1 = \text{do\_nothing}$, $Z_1 = \text{door}$

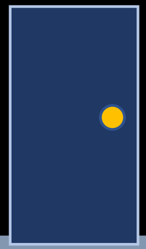
State	0	1	2	3	4	5
$p(x_1)$	0	0	0	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{1}{3}$

At $t = 2$, $U_2 = -1$, $Z_2 = \text{door}$

State	0	1	2	3	4	5
$p(x_2)$	0	0	0	$\frac{1}{6} \times 0$	$\frac{\frac{1}{2} \times \frac{1}{4}}{\frac{1}{2} \times \frac{1}{4} + \frac{1}{3} \times \frac{1}{2}}$	$\frac{\frac{1}{3} \times \frac{1}{2}}{\frac{1}{2} \times \frac{1}{4} + \frac{1}{3} \times \frac{1}{2}}$

State	0	1	2	3	4	5
$p(x_2)$	0	0	0	0	$\frac{3}{7}$	$\frac{4}{7}$

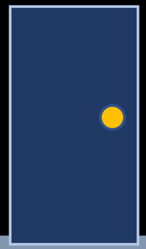
Bayes Filter - Example 1 (initial conditions 1)



At t=0, we are absolutely certain the robot is at state $X_0 = 0$

State	0	1	2	3	4	5
$p(x_0)$						

Bayes Filter - Example 1 (initial conditions 1)



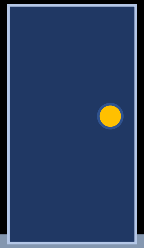
At t=0, we are absolutely certain the robot is at state $X_0 = 0$

State	0	1	2	3	4	5
$p(x_0)$	1	0	0	0	0	0

At t=1, U_1 = do_nothing , Z_1 = door

State	0	1	2	3	4	5
$p(x_1)$						

Bayes Filter - Example 1 (initial conditions 1)



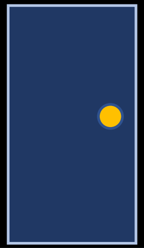
At t=0, we are absolutely certain the robot is at state $X_0 = 0$

State	0	1	2	3	4	5
$p(x_0)$	1	0	0	0	0	0

At t=1, U_1 = do_nothing , Z_1 = door

State	0	1	2	3	4	5
$p(x_1)$	0	0	0	0	0	0

Bayes Filter - Example 1 (initial conditions 2)



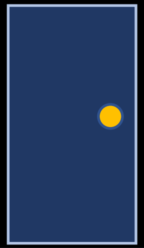
At t=0, we are "absolutely" certain the robot is at state $X_0 = 0$

State	0	1	2	3	4	5
$p(x_0)$	$\frac{19}{20}$	$\frac{1}{100}$	$\frac{1}{100}$	$\frac{1}{100}$	$\frac{1}{100}$	$\frac{1}{100}$

At t=1, $U_1 = \text{do\_nothing}$, $Z_1 = \text{door}$

State	0	1	2	3	4	5
$p(x_1)$						

Bayes Filter - Example 1 (initial conditions 2)



At t=0, we are “absolutely” certain the robot is at state $X_0 = 0$

State	0	1	2	3	4	5
$p(x_0)$	$\frac{19}{20}$	$\frac{1}{100}$	$\frac{1}{100}$	$\frac{1}{100}$	$\frac{1}{100}$	$\frac{1}{100}$

At t=1, U_1 = do_nothing , Z_1 = door

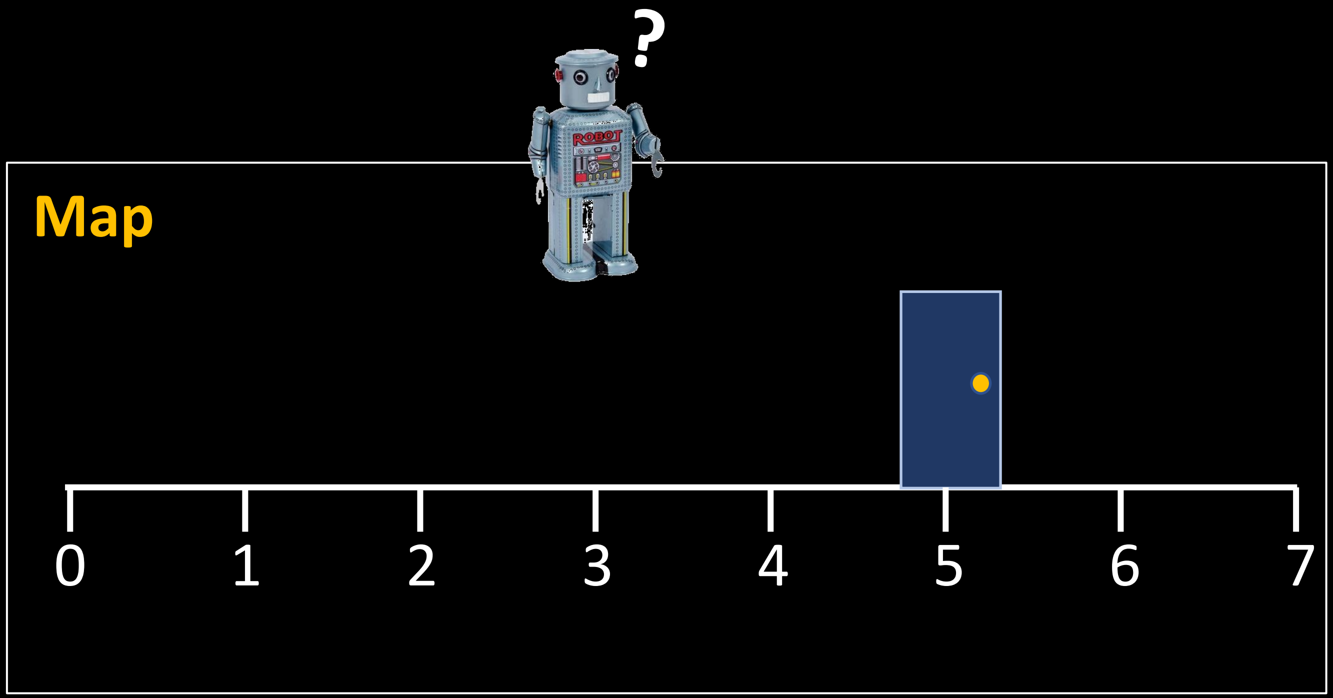
State	0	1	2	3	4	5
$p(x_0)$	0	0	0	0	$\frac{1}{3}$	$\frac{2}{3}$

Always believe, even if just a little, in the improbable!

(deterministic approaches are fragile!)

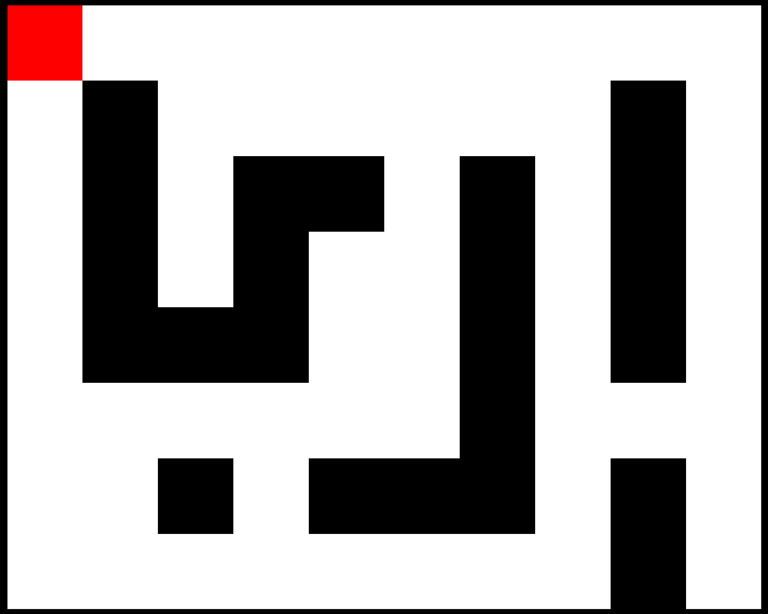
Bayes Filter – Example 2

- Bayes with beans
  - World
    - 1D continuous robot world
    - Discretized into 7 states
    - …with a door at state 5
  - Motion model
    - 80% correct, 20% fails
  - Sensor model
    - 90% correct, 10% fails
  - Initial belief
  - Take an action: +1
  - Take a sensor reading: door!



Bayes Filter – Example 3

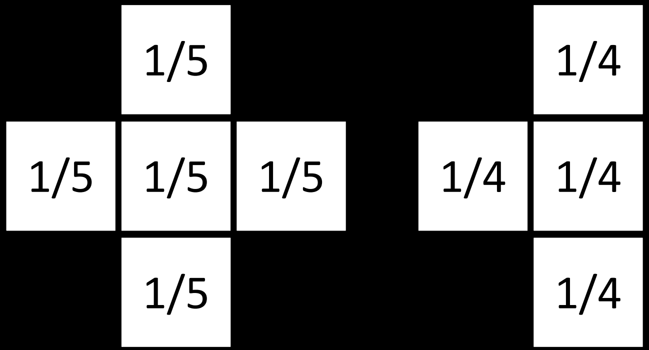
- 8x10 discrete world
  - Known map with obstacles and walls
- Robot state
  - Location in the map (no orientation)
  - Initial state is (0,0)

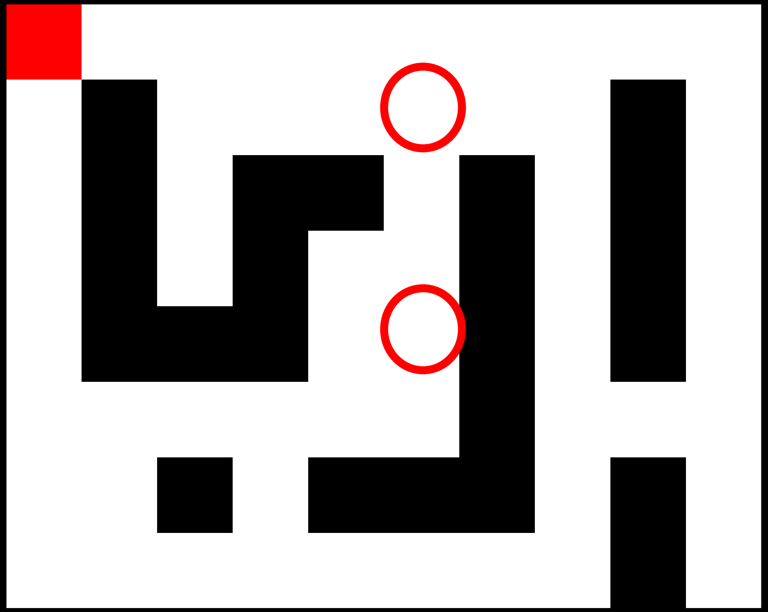


X is the set of possible locations
x is one of these locations

Bayes Filter – Example 3

- Transition model
  - No matter what I tell my robot to do, it makes a random move or stays in place!
  - E.g.

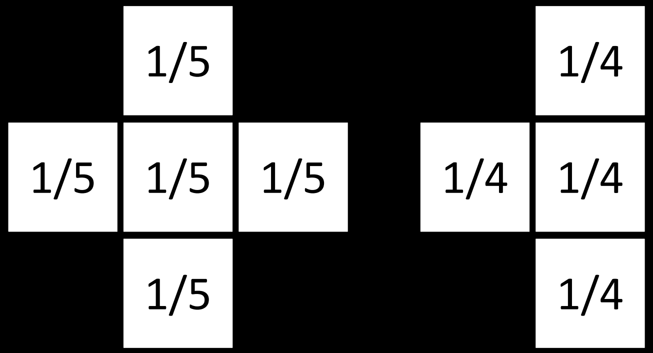


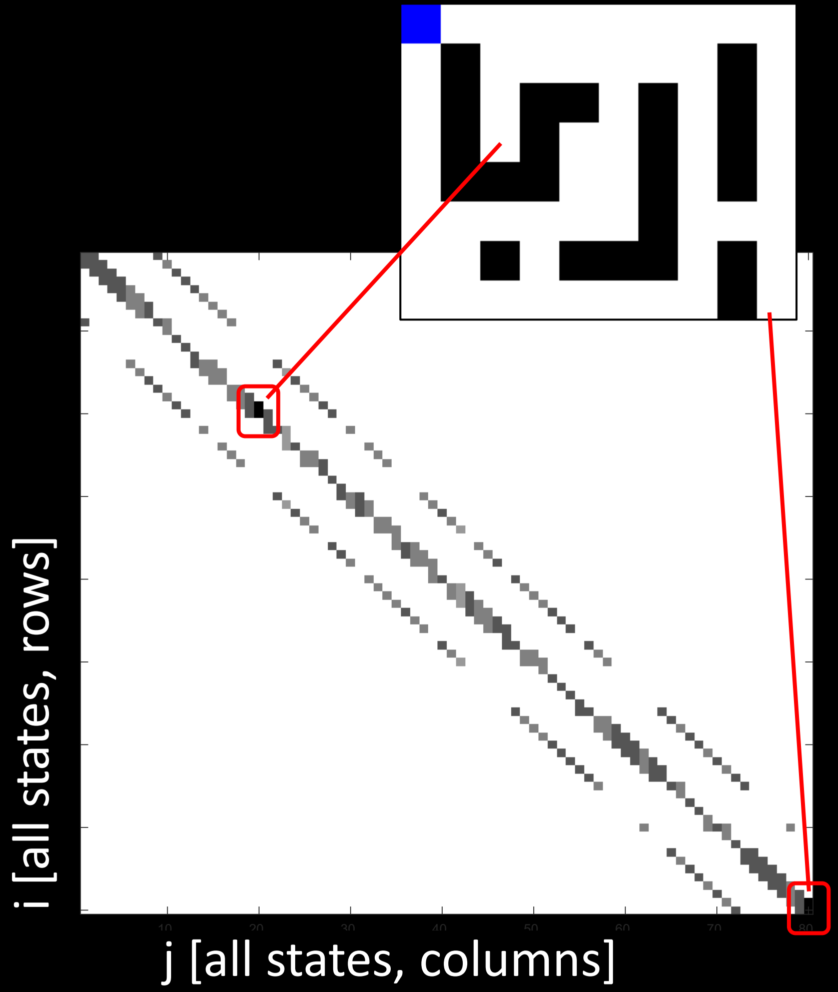


X is the set of possible locations
x is one of these locations

Bayes Filter – Example 3

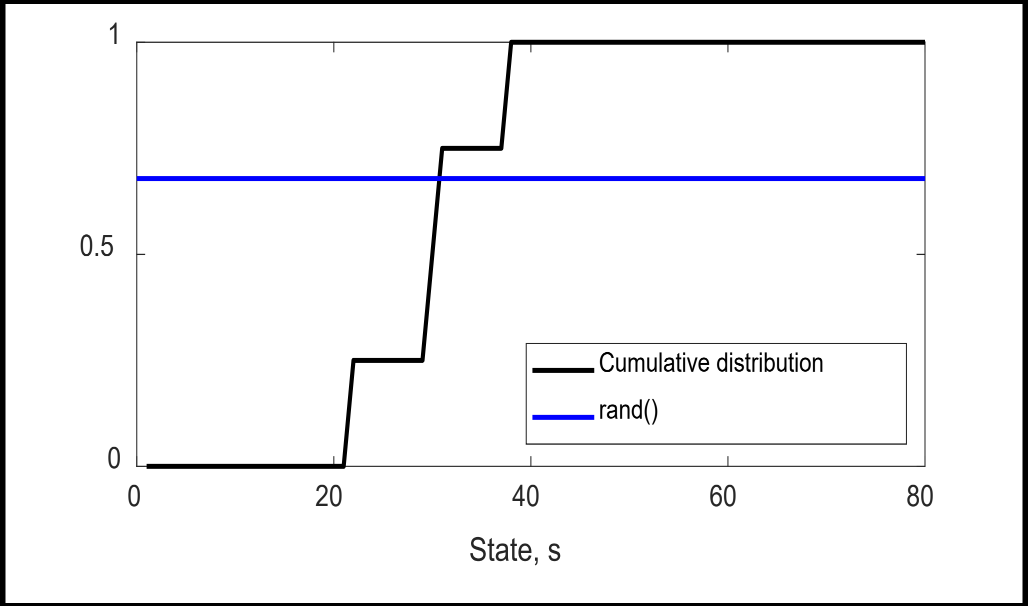
- Transition model
  - No matter what I tell my robot to do, it makes a random move or stays in place!
  - Transition matrix, A
    - Probability to move from state *j* to state *i*

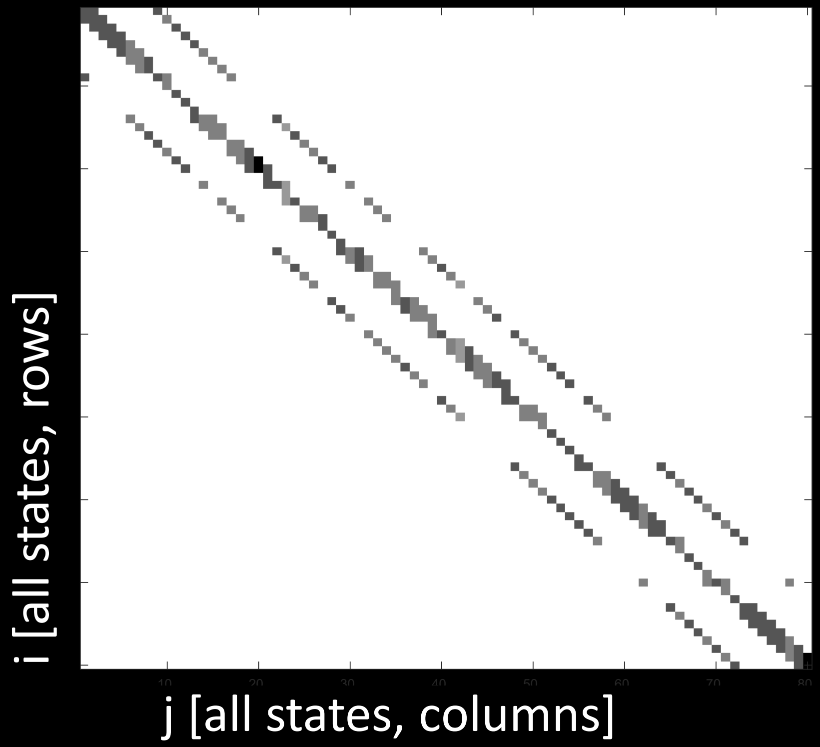




Bayes Filter – Example 3

- Practical implementation
  - Set up our world
  - Compute the transition matrix, A
  - Take actions
    - Cumulative distribution
    - `find(Mtri*A*s >= rand(),1,'first');`





Bayes Filter – Example 3

- Prediction step

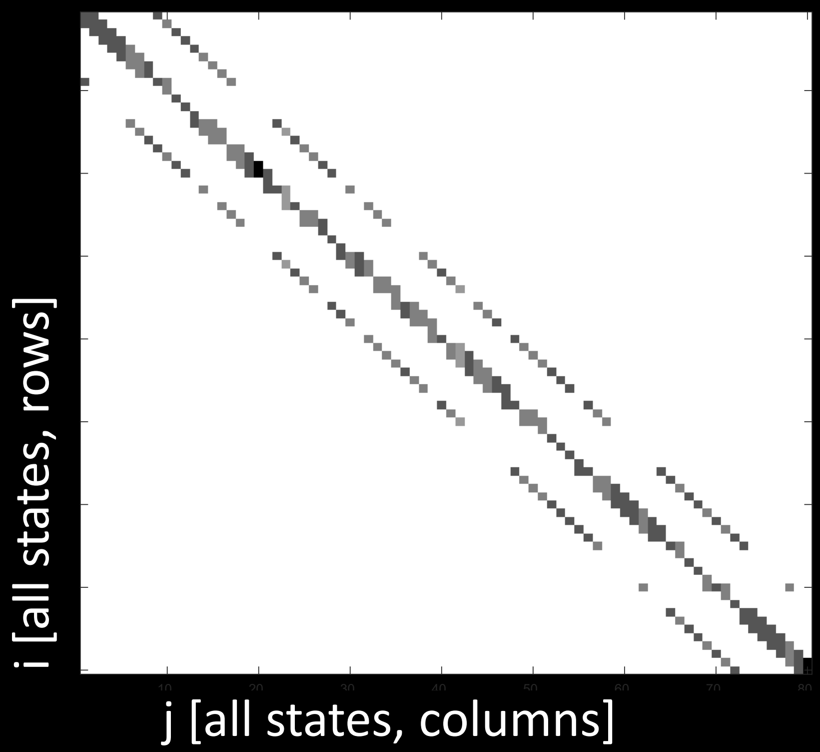
Prediction step $(bel(x_{t-1}), \cancel{u_t})$:

1. for all x_t do
2. $\overline{bel}(x_t) = \sum_{x_{t-1}} p(x_t | \cancel{u_t}, x_{t-1}) \; bel(x_{t-1})$
3. endfor

Matrix implementation

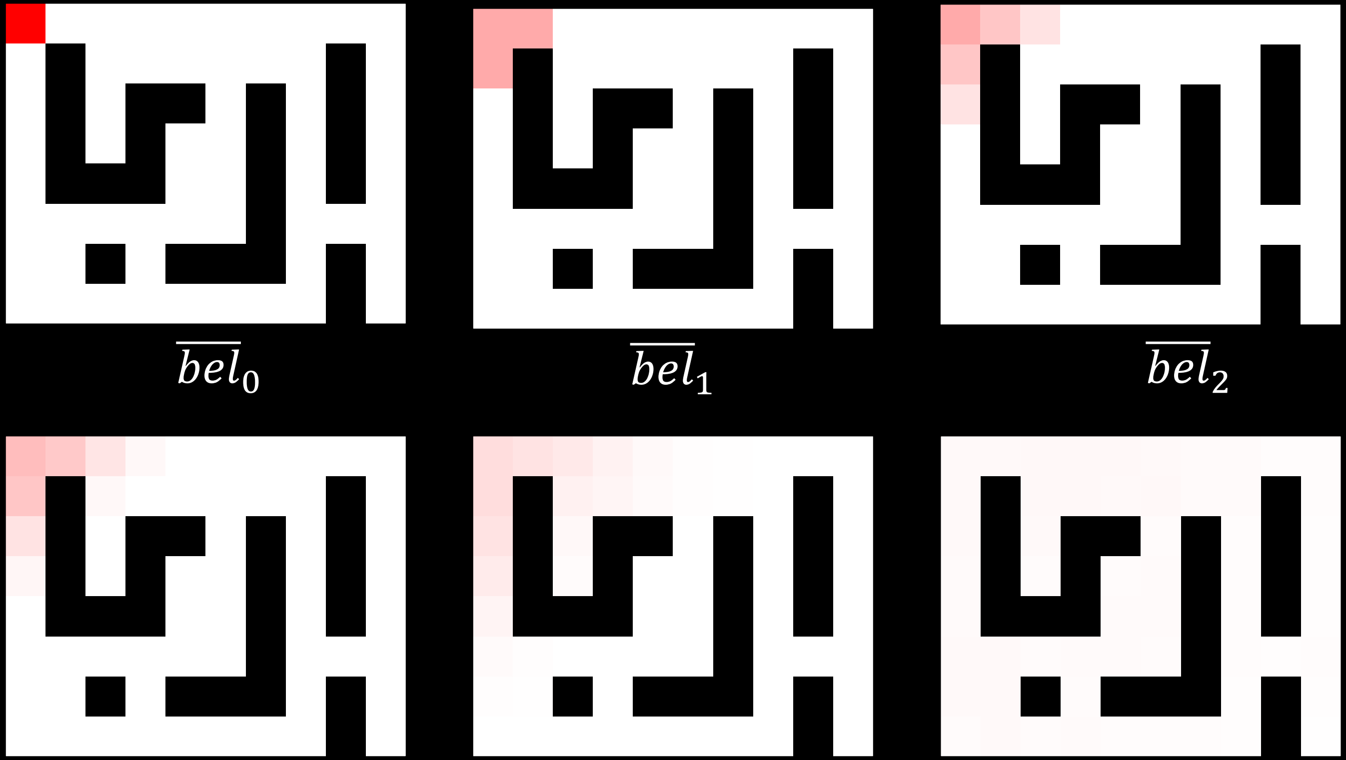
1. $\overline{bel} = A \; bel_{t-1}$

…where A is the transition matrix (80x80) and bel is the probability distribution over all states (80x1)



Bayes Filter – Example 3

- Prediction step

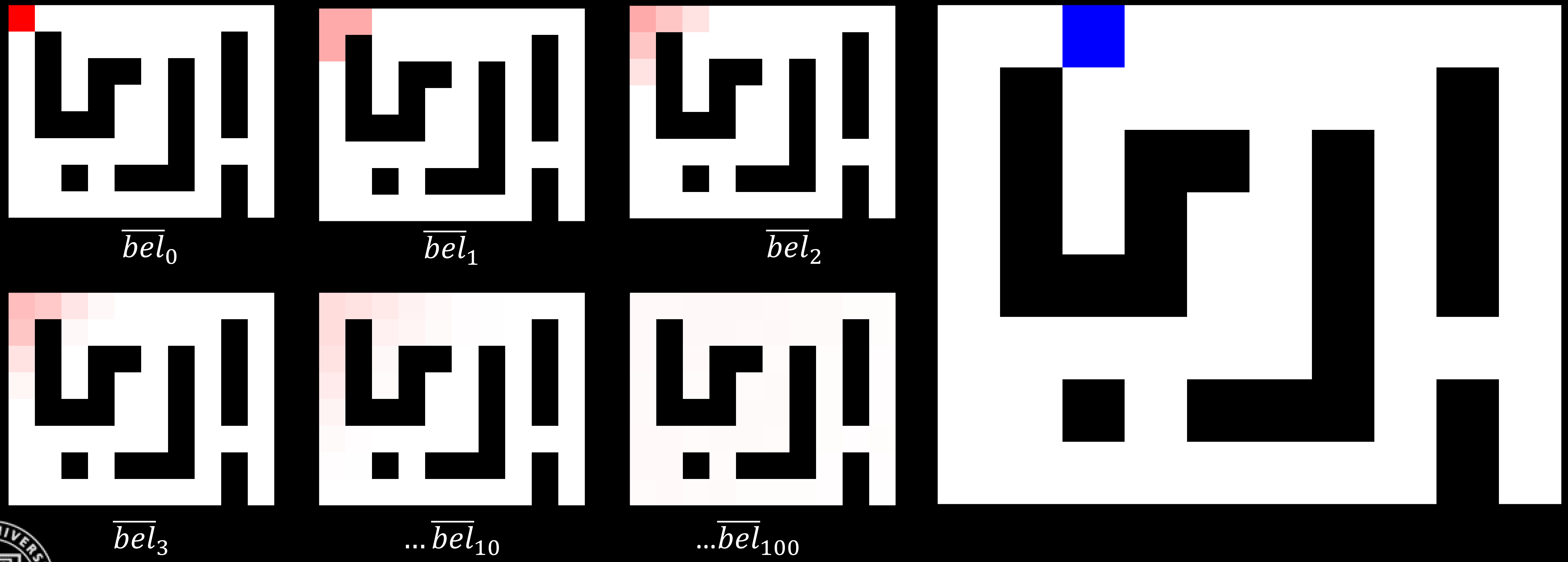


$\overline{bel}_0$ $\overline{bel}_1$ $\overline{bel}_2$

$\overline{bel}_3$ $\ldots\overline{bel}_{10}$ $\ldots\overline{bel}_{100}$

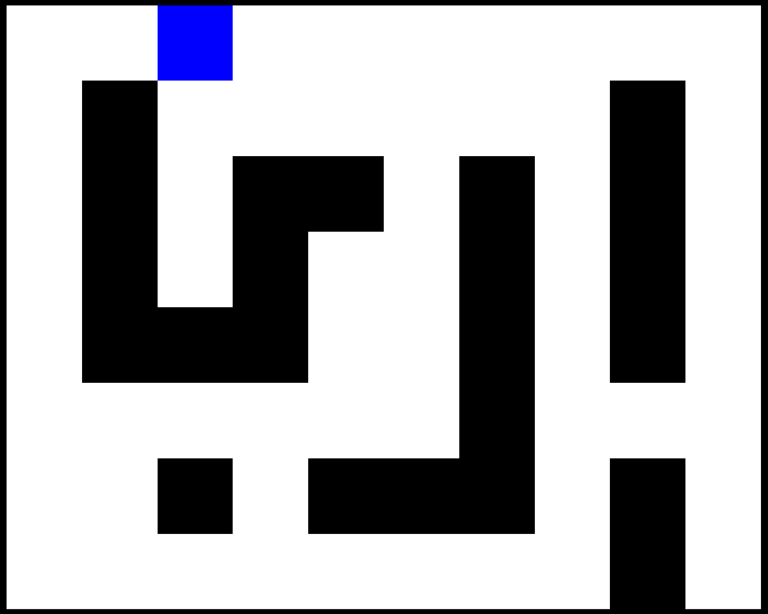
Bayes Filter – Example 3

- The robot may not know where it is, but it *does* have a physical state
- And it will have observations tied to that state



Bayes Filter – Example 3

- Observation model
  - In every time step, we sense each of the four neighboring cells [N, E, S, W]
  - In *z*, each reading is independent and correct with 90% probability

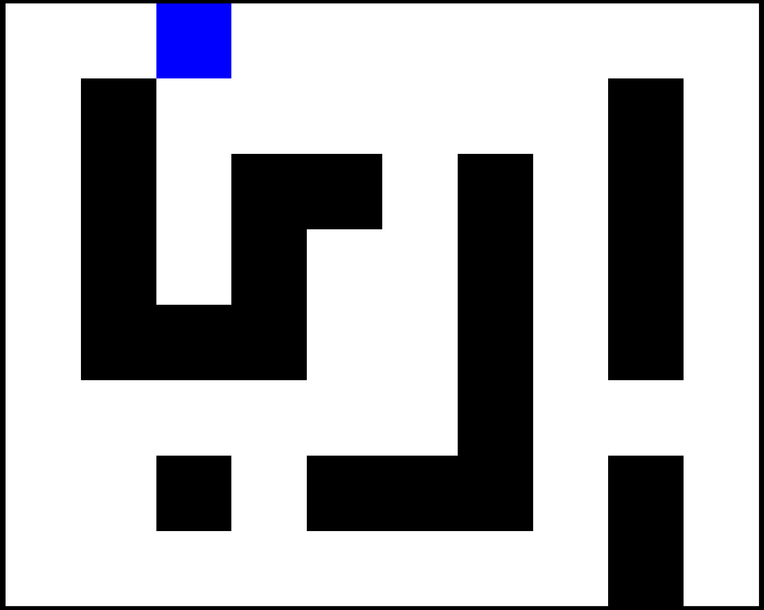


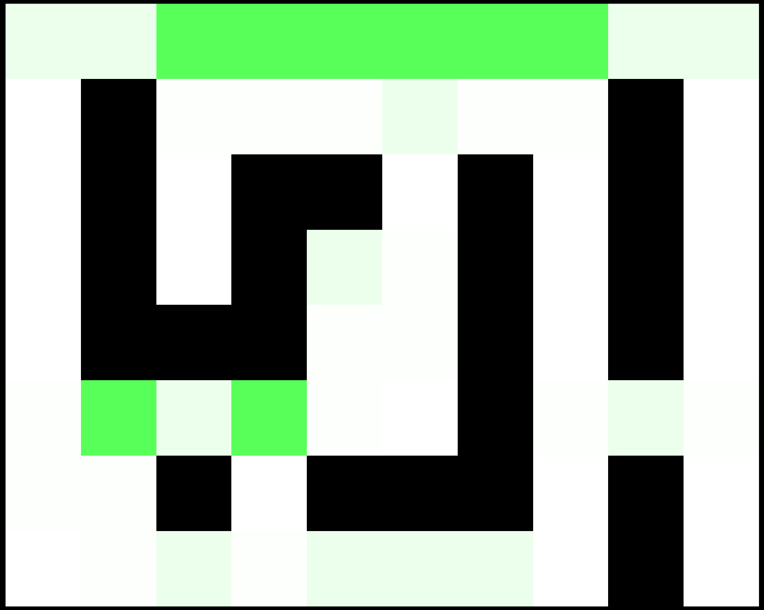
X is the set of possible locations
x is one of these locations
z are the sensor measurements

Bayes Filter – Example 3

- Observation model
  - In every time step, we sense each of the four neighboring cells [N, E, S, W]
  - In *z*, each reading is independent and correct with 90% probability

- `P(no walls | x) = 0.1*0.9*0.9*0.9`
- `P(N        | x) = 0.9*0.9*0.9*0.9` ← *highest likelihood*
- `P(W        | x) = 0.1*0.9*0.9*0.1`
- `P(S        | x) = 0.1*0.9*0.1*0.9`
- `P(E        | x) = 0.1*0.1*0.9*0.9`
- …
- `P(NW       | x) = 0.9*0.9*0.9*0.1`
  - How many combinations are there per state?
    - 2^4





P(z|X)

Bayes Filter – Example 3

- Observation model
  - In every time step, we sense each of the four neighboring cells [N, E, S, W]
  - In z, each reading is independent and correct with 90% probability

- If all readings are correct:
  - $\sum|z_t - z'_{xt}| = 0$
  - $p_z(x_t)$=0.6561
- If all readings are wrong:
  - $\sum|z_t - z'_{xt}| = 4$
  - $p_z(x_t)$=0.0001

Algorithm Bayes_Filter $(bel(x_{t-1}), u_t, z_t)$:

1. for all x_t do
2. $\overline{bel}(x_t) = \sum_{x_{t-1}} p(x_t|u_t, x_{t-1})\ bel(x_{t-1})$
3. $bel(x_t) = \eta\, p(z_t|x_t)\ \overline{bel}(x_t)$
4. endfor
5. return $bel(x_t)$

Compute likelihood of observations, p_{zX}

1. for all x_t do
2. $p_{zX}(x_t) = 0.9^{4-\sum|z_t - z'_{xt}|}\ 0.1^{\sum|z_t - z'_{xt}|}$
3. Endfor

…where p_{zX} is a vector (80x1)

Bayes Filter – Example 3

- Observation model
  - In every time step, we sense each of the four neighboring cells [N, E, S, W]
  - In z, each reading is independent and correct with 90% probability

Algorithm Bayes_Filter $(bel(x_{t-1}), u_t, z_t)$:

1. for all x_t do
2. $\overline{bel}(x_t) = \sum_{x_{t-1}} p(x_t|u_t, x_{t-1})\ bel(x_{t-1})$
3. $bel(x_t) = \eta\, p(z_t|x_t)\ \overline{bel}(x_t)$
4. endfor
5. return $bel(x_t)$

Compute new belief

1. $bel_t = p_{zX}\ \overline{bel}\ / \sum(p_{zX}\ \overline{bel})$

…where $\overline{bel}$ is a vector (80x1)
and p_{zX} is a vector (80x1)

Bayes Filter – Example 3

- Bayes Filter

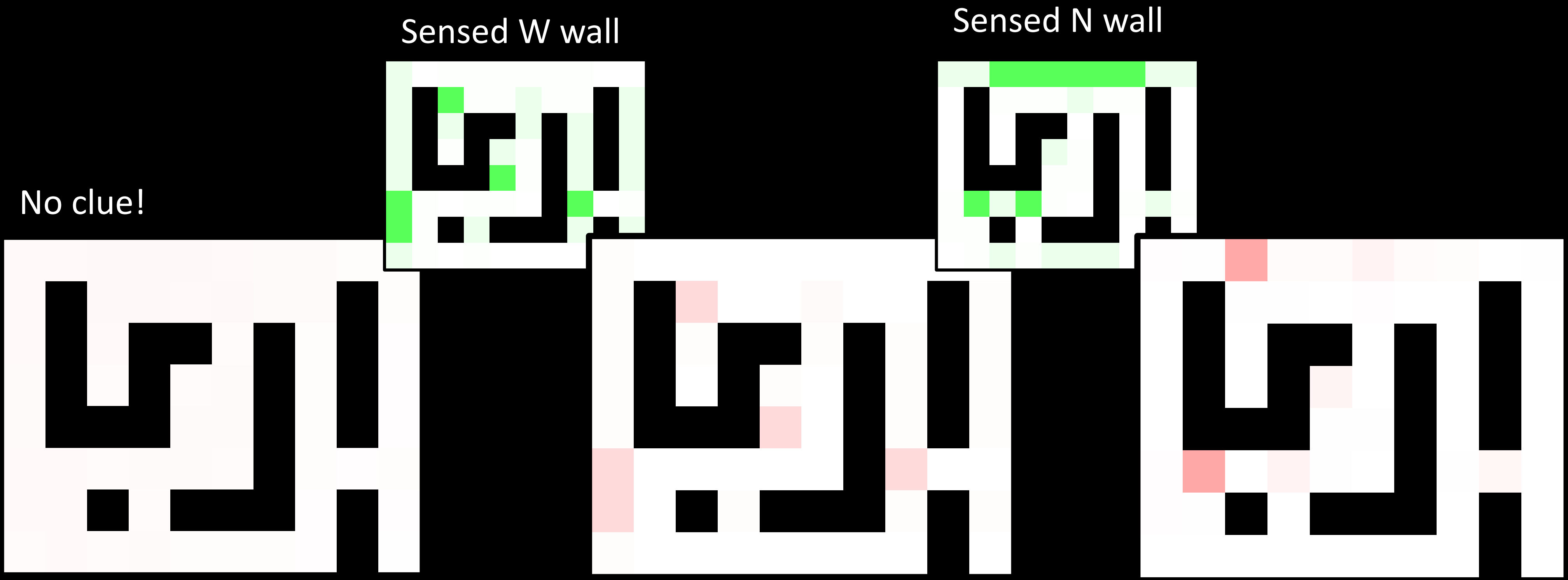
Algorithm Bayes_Filter(bel_{t-1}, z_t):

1. $\overline{bel} = A\, bel_{t-1}$
2. for all x_t do
3. $p_{zX}(x_t) \;=\; 0.9^{4-\sum|z_t - z'_{xt}|}\; 0.1^{\sum|z_t - z'_{xt}|}$
4. end for
5. $bel_{\mathrm{t}} \;=\; \overline{bel}\, p_{zX} \,/\, \sum(\overline{bel}\, p_{zX})$

Only do computations for states with a belief > threshold

Precache and look up for faster operation

Bayes Filter – Example 3



In two steps, we homed in on where we are!
…

- ***How good is the Bayes Filter?***
- ***Can you do better?***
  - Improved transition model
  - Deliberately move in directions that give you more information

Bayes Filter II

Algorithm Bayes_Filter $(bel(x_{t-1}), u_t, z_t)$:

1. for all x_t do
2. $\overline{bel}(x_t) = \sum_{x_{t-1}} p(x_t|u_t, x_{t-1})\ bel(x_{t-1})$ [Prediction Step]
3. $bel(x_t) = \eta\, p(z_t|x_t)\ \overline{bel}(x_t)$ [Update/Measurement Step]
4. endfor
5. return $bel(x_t)$

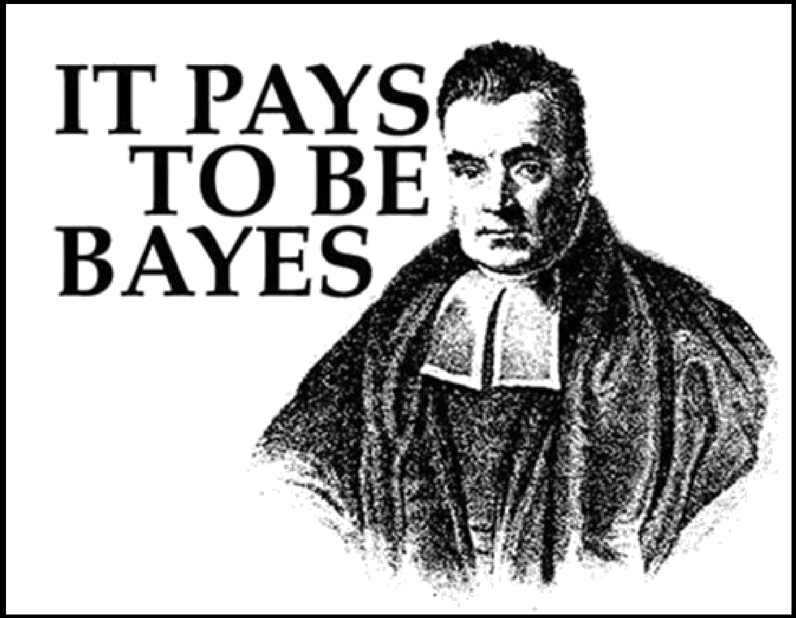
- Example 1
  - Robot in a 1D world
  - The importance of having some belief in all states
- Example 2
  - Bayes with beans
  - The importance of normalization
- Example 3
  - (x,y)-robot in a grid world
  - Computational efficiency
    - Matrices
    - Pre-cache observations

Summary

- Use temporal consistency between observations that are poor estimates individually
- Localization can work with…
  - …completely random motion
  - …noisy sensors
  - Remember to…
    - Don't be deterministic
    - Normalize
    - Efficient computation

Algorithm Bayes_Filter $(bel(x_{t-1}), u_t, z_t)$:

1. for all x_t do
2. $\overline{bel}(x_t) = \sum_{x_{t-1}} p(x_t|u_t, x_{t-1})\ bel(x_{t-1})$
3. $bel(x_t) = \eta\, p(z_t|x_t)\ \overline{bel}(x_t)$
4. endfor
5. return $bel(x_t)$



ECE 4160/5160
MAE 4910/5910

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Fast Robots

Flipped Classroom 4/13/23 (*Thursday!*)

Please install the simulator

https://cei-lab.github.io/FastRobots-2023/FastRobots-Sim.html